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Burak Ünveren

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Burak Ünveren[1]

Yıldız Teknik Üniversitesi, İİBF, Davutpaşa Campus, İstanbul, Turkey

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ABSTRACT

Purpose:
The purpose of this paper is to analyze the efficiency loss due to incomplete financial markets when risk is induced by technological uncertainty.
Design/methodology/approach:
A worker-capitalist general equilibrium model is developed. It is assumed that future technical change is a stochastic event, causing uncertainty in future relative prices. Then the model is calibrated to the US data.
Findings:
Our first finding is theoretical: the competitive equilibrium is Pareto-inefficient. Then we numerically calculate the taxes that make all individuals better-off at the calibrated parameter values. The results clearly show how the burden of taxation should be shared among workers and capitalists when the government uses redistribution of income as a tool of mitigating the loss of efficiency due to technological shocks.
Research limitations/implications:
The model is obviously a stripped-down version of reality, and hence, the results should be taken with a grain of salt as the numerical computations would be definitely sensitive to certain rich details of real life that are neglected in this study.
Originality/value:
The results show that the total amount of employment, and production are not affected by optimal taxation, which is a surprising result. Indeed, the inefficiency is primarily caused by the distribution of labor supply among individuals. The optimal taxes are also numerically computed.

1. Introduction

Innovation ensures the feasibility of improving the welfare of all individuals in a given society. Nevertheless, technical change is seldom unanimously supported. For example, labor saving technology poses a serious risk to the welfare of blue collar workers since they often cannot insure themselves against the possibility of lower wages caused by changes in technology. That is why, a labor saving technical change can be resisted by manual workers due to the possibility of a fall in wages.

This study considers a similar scenario in a general equilibrium setup. We assume technical progress is a stochastic event causing uncertainty in future relative prices. To the best of our knowledge, no market economy offers an insurance against unfavorable relative prices caused by future technical change. Lack of insurance is known as incomplete markets in economic theory.

The most fundamental problem inflicted by market incompleteness is that competitive equilibrium may fail to be Pareto-efficient. Of course, this does not immediately command government intervention. Economic policies, regardless of how genuinely designed they may be, can also fail to bring about efficiency if the government is also subject to the same incompleteness of markets that all other agents face. Indeed, Diamond (1967) proves that is exactly the case when there is a single good in every possible state of the economy.
Interestingly, Diamond's result does not generalize to economies with multiple goods. For example, assuming there are multiple goods, Citanna *et. al.* (1998, 2006) show that competitive equilibria are generically constrained Pareto-inefficient, which means government intervention subject to the same constraints that individuals face almost certainly brings about higher utility for some individuals without hurting anyone in equilibrium.

In this study, we study a two-period production economy with workers and capitalists who are otherwise identical. The second period involves the possibility of a labor-saving technological progress with a given probability. This potential of labor-saving technology poses the risk of lower employment and wages for workers. Since there is no insurance for the possibility of lower real wages in real life, we also assume that individuals cannot insure themselves against the risk of

[1]Corresponding Author: Burak Ünveren
Email :bunveren@yildiz.edu.tr.
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possible changes in relative prices in the future. Our first result theoretically shows that under these conditions the competitive equilibrium is not Pareto-efficient. Then we numerically analyze, by calibrating the model parameters to the US data, whether the redistribution of income can ensure higher utility for workers without hurting the capitalists.

The simulation results suggest that the redistribution of income is capable of restoring full Pareto-efficiency. Furthermore, we also show that this can be achieved via two different methods which yield identical results. According to the first approach, workers should be taxed in the first period to finance the subsidies that they would get in case the labor saving technological progress actually takes place. The second approach stipulates taxing the capitalists in the first period to finance their subsidies in the second period if no technological progress takes place.

It is noteworthy that the results also show that the employment, and thus production, with redistribution of income is identical to the case of no intervention Indeed, the difference between two polar cases is the labor supply decisions of workers and capitalists while total employment is the same. Workers supply more and capitalists supply less labor when there is no government intervention (implying inefficiency) compared to the case of optimally designed redistribution of income.

The next section introduces the model. Calibration results are presented in Section 3. Section 4 is the conclusion.

2. The Model

Consider a two-period economy with n individuals and m firms. Individuals enjoy consuming a produced good and leisure in each period. In the first period there is no uncertainty. However, in the second period, there are two possible states of the world that can be realized. The uncertainty is due to an exogenous technical change.

More formally, there are three states denoted by $s = 0,1,2$. The state 0 denotes the first period where there is no uncertainty. The states 1 and 2 are two possible states of the world in the second period. Suppose that state s will occur with probability θ_s. Since the first period (i.e. state 0) involves no uncertainty, it follows that $\theta_0 = 1$. It is also certain that either state 1 or 2 occurs in the second period, implying $\theta_1 + \theta_2 = 1$. Write $\theta = (\theta_0, \theta_1, \theta_2)$.

The only exogenous of the model depending on s is the production function

$$f_s(L_{j,s})$$

where $L_j = (L_{j,0}, L_{j,1}, L_{j,2})$ is a vector denoting the labor demand by firm j for all possible states of the economy. Assume that $f_s(\cdot)$ is concave and smooth. Observe that the production technology $f_s(\cdot)$ depends on the state of the world s, whose details will be made explicit in the sequel.

Write p_s for the price of the produced consumption good, and w_s for the wage of labor in state $s = 0,1,2$. Then, given the production technology, prices, wages, and the probability distribution of possible states, each firm j maximizes expected total profits by solving

$$\max_{L_j} \sum_{s=0,1,2} \theta_s \pi_{j,s} \tag{1}$$

where

$$\pi_{j,s} = p_s f_s(L_{j,s}) - w_s L_{j,s}$$

is the profit generated by firm j in state $s = 0,1,2.$
Profits of the firms are distributed to their shareholders. The sum of profit income that individual i receives is denoted by $\pi_{i,s}$ for each state $s = 0,1,2$. There are two types of individuals. Workers, the first type of individuals, have no profit share, and thus, finance their consumption only by supplying labor as we shall see soon. Property owners (i.e. capitalists), the second type of individuals, own strictly positive amounts of profit shares. Assume that all property owners own equal profit shares.

All workers and property owners are endowed with preferences represented by a concave and smooth utility function $U_i(c_i, l_i)$ where $c_i = (c_{i,0}, c_{i,1}, c_{i,2})$ and $l_i = (l_{i,0}, l_{i,1}, l_{i,2})$ represent the consumption good and leisure enjoyed by the individual i in states $(0,1,2)$.

The preferences of the individual can be represented in expected utility form:

$$U_i(c_i, l_i) = \sum_{s=0,1,2} \theta_s u_i(c_{i,s}, l_{i,s}) \tag{2}$$

where u_i is the instantaneous utility function. Suppose that

$$u_i(c_{i,s}, l_{i,s}) = \frac{c_{i,s}^{1-\sigma}}{1-\sigma} + \mu \frac{l_{i,s}^{1-\sigma}}{1-\sigma}$$

implying the elasticity of substitution between consumption and leisure $1/\sigma$ is constant. The parameter μ gives the relative weight of utility derived from leisure with respect to utility derived from consumption.

The asset markets are assumed to be incomplete. This means there is no way to insure consumption between possible states 1 and 2 in the second period. Therefore, the budget constraint is given by

$$\beta_i(p, w, q) = \left\{ (c_i, l_i, z_i): \begin{array}{c} p_0 c_{i,0} + w_0 l_{i,0} + q z_i = w_0 + \pi_{i,0} \\ p_s c_{i,s} + w_s l_{i,s} = z_i + w_s + \pi_{i,s} \\ for\ each\ s = 1,2 \end{array} \right\}$$

where q is the price of financial assets, and z_i is the amount of financial assets bought by individual i.

Under these conditions, the individual i maximizes expected utility by solving

$$\max_{(c_i, l_i, z_i)} U_i(c_i, l_i) \ s.t. (c_i, l_i) \in \beta_i(p, w, q)$$

where $\beta_i(p, w, q)$ is the budget of individual i.

Notice that all exogenous of the model except the production technology (e.g. preferences, endowments, etc.) are certain. The markets are incomplete since there is no insurance against the uncertainty inherent in the production technology.

Now the definition of competitive equilibrium with incomplete markets (CEI) can be presented:

Definition 1: CEI is a vector $\langle (c_i, l_i, z_i)_{i=1}^n, (L_j)_{j=1}^n, q, (p_s, w_s)_{s=0,1,2} \rangle$ such that

(c_i, l_i, z_i) solves (1) for each i, and L_j solves (2) for each j, and product, labor, and asset markets clear:

$$\sum_i c_{i,s} - \sum_j f_s(L_{j,s}) = 0$$

$$\sum_i l_{i,s} + \sum_j L_{j,s} - n = 0$$

$$\sum_i z_{i,s} = 0$$

for each $s = 0,1,2$.

This equilibrium definition captures the notion of competitiveness in the sense that consumers and firms are price takers, and markets clear. However, competitiveness does not suffice for Pareto-efficiency in the present context in contrast to the competitive equilibrium *à la* Arrow-Debreu-McKenzie. That is to say, CEI may not be Pareto-efficient. Now let us see that this is actually the case.

Theorem 1: CEI is not Pareto-efficient.

Proof: Let

$$\langle (c_i, l_i, z_i)_{i=1}^n, (L_j)_{j=1}^n, q, (p_s, w_s)_{s=0,1,2} \rangle$$

denote the CEI. First note that there are 3 Walras' Laws in this economy implying we need to normalize prices three times. Let $p_s = 1$ for all $s = 0,1,2$ without loss of generality.

The first order conditions of individual optimality are

$$\frac{du_i}{dc_{i,s}} - \lambda_{i,s} = 0 \qquad (3)$$

$$\frac{du_i}{dl_{i,s}} - \lambda_{i,s} w_s = 0$$

$$q\lambda_{i,0} - \lambda_{i,1} - \lambda_{i,2} = 0$$

for each $s = 0,1,2$, and

$$c_{i,0} + w_0 l_{i,0} + q z_i = w_0 + \pi_{i,0} \qquad (4)$$

$$c_{i,1} + w_1 l_{i,1} = z_i + w_1 + \pi_{i,1}$$

$$c_{i,2} + w_2 l_{i,2} = z_i + w_2 + \pi_{i,2}$$

at the CEI since (c_i, l_i, z_i) solves individual i's optimization problem. As usual, $\lambda_{i,s}$ is the Lagrange multiplier associated with the budget constraint of individual i at state s.

Had the CEI been Pareto-efficient, the equilibrium allocation $(c_i, l_i)_{i=1}^n$ and $(L_j)_{j=1}^m$ would also solve

$$\max \sum_i \rho_i U_i\,(c_i, l_i)$$

s.t.

$$\sum_i c_{i,s} - \sum_j f_s(L_{j,\square}) = 0$$

$$\sum_i l_{i,s} + \sum_j L_{j,s} - n = 0$$

for some positive welfare weight $\rho = (\rho_1, \ldots, \rho_n)$. The first order conditions are

$$\rho_i \frac{du_i}{dc_{i,s}} - \gamma_s = 0 \qquad (5)$$

$$\rho_i \frac{du_i}{dl_{i,s}} - \delta_s = 0$$

$$\delta_s - \gamma_s f_s{}'(L_{j,s}) = 0$$

First let us see that $\rho = (\rho_1, \ldots, \rho_n)$ is proportional to

$$\left(\frac{1}{\lambda_{1,s}}, \cdots, \frac{1}{\lambda_{n,s}}\right).$$

To see that, first note that if $\rho = (\rho_1, \ldots, \rho_n)$ is a vector of welfare weights solving the Pareto-efficiency conditions (3-4), then $a\rho = (a\rho_1, \ldots, a\rho_n)$ is also an admissible vector of welfare weights for any $a > 0$. This means one of the welfare weights, say ρ_1, can be set to any arbitrary positive number.

Hence, let

$$\rho_1 = 1$$

which implies

$$\gamma_s = \lambda_{1,s}, s = 0,1,2.$$

As a consequence,

$$\delta_s = w_s \lambda_{1,s}$$

and

$$(\rho_1, \ldots, \rho_n) = \left(1, \frac{\lambda_{1,s}}{\lambda_{2,s}}, \cdots, \frac{\lambda_{1,s}}{\lambda_{n,s}}\right).$$

due to (3-5). Deduce that the marginal rates of substitution between any two goods at any states are equal for all individuals. This is the standard condition of equal marginal rates of substitution among individuals for Pareto-efficiency.

It follows that

$$\frac{\frac{du_1}{dc_{1,1}}}{\frac{du_1}{dc_{1,s}}} = \frac{\frac{du_i}{dc_{i,1}}}{\frac{du_i}{dc_{i,s}}}$$

and

$$\frac{\frac{du_1}{dc_{1,1}}}{\frac{du_1}{dl_{1,s}}} = \frac{\frac{du_i}{dc_{i,1}}}{\frac{du_i}{dl_{i,s}}}$$

for all i and s. In open form,

$$\left(\frac{c_{1,1}}{c_{1,s}}\right)^{-\sigma} = \left(\frac{c_{i,1}}{c_{i,s}}\right)^{-\sigma} \text{ and } \left(\frac{c_{1,1}}{l_{1,s}}\right)^{-\sigma} = \left(\frac{c_{i,1}}{l_{i,s}}\right)^{-\sigma}$$

which is equivalent to

$$\frac{c_{1,1}}{c_{1,s}} = \frac{c_{i,1}}{c_{i,s}} \text{ and } \frac{c_{1,1}}{l_{1,s}} = \frac{c_{i,1}}{l_{i,s}}.$$

In other words, k is such that $(c_1, l_1) = k \times (c_i, l_i)$. Assume, without loss of generality, individual 1 is a worker, and individual i is a property owner. This implies

$$c_{1,0} + w_0 l_{1,0} + q z_1 = w_0$$

$$c_{1,1} + w_1 l_{1,1} = z_1 + w_1$$

$$c_{1,2} + w_2 l_{1,2} = z_1 + w_2$$

and

$$k(c_{1,0} + w_0 l_{1,0}) + q z_i = w_0 + \pi_{i,0}$$

$$k(c_{1,1} + w_1 l_{1,1}) = z_i + w_1 + \pi_{i,1}$$

$$k(c_{1,2} + w_2 l_{1,2}) = z_i + w_2 + \pi_{i,2}.$$

However, the first order conditions of individual 1 given by (3) yields

$$c_{1,s} = (w_s)^{\frac{1}{\sigma}} l_{1,s}$$

for all s. Therefore

$$l_{1,0}\left(w_0 + (w_0)^{\frac{1}{\sigma}}\right) + q z_1 = w_0$$

$$l_{1,1}\left(w_1 + (w_1)^{\frac{1}{\sigma}}\right) = z_1 + w_1$$

$$l_{1,2}\left(w_2+(w_2)^{\frac{1}{\sigma}}\right)=z_1+w_2$$

and

$$kl_{1,0}\left(w_0+(w_0)^{\frac{1}{\sigma}}\right)+qz_i=w_0+\pi_{i,0}$$
$$kl_{1,1}\left(w_1+(w_1)^{\frac{1}{\sigma}}\right)=z_i+w_1+\pi_{i,1}$$
$$kl_{1,2}\left(w_2+(w_2)^{\frac{1}{\sigma}}\right)=z_i+w_2+\pi_{i,2}.$$

As a consequence, observe that

$$q=\frac{w_0-l_{1,0}\left(w_0+(w_0)^{\frac{1}{\sigma}}\right)}{l_{1,1}\left(w_1+(w_1)^{\frac{1}{\sigma}}\right)-w_1}$$
$$z_1=l_{1,1}\left(w_1+(w_1)^{\frac{1}{\sigma}}\right)-w_1$$
$$l_{1,2}\left(w_2+(w_2)^{\frac{1}{\sigma}}\right)=l_{1,1}\left(w_1+(w_1)^{\frac{1}{\sigma}}\right)-w_1+w_2.$$

Since

$$z_i=-\frac{W}{K}z_1$$

due to market clearing in the financial market,

$$kl_{1,0}\left(w_0+(w_0)^{\frac{1}{\sigma}}\right)-\frac{W}{K}\left(w_0-l_{1,0}\left(w_0+(w_0)^{\frac{1}{\sigma}}\right)\right)=w_0+\pi_{i,0}$$
$$kl_{1,1}\left(w_1+(w_1)^{\frac{1}{\sigma}}\right)=-\frac{W}{K}\left(l_{1,1}\left(w_1+(w_1)^{\frac{1}{\sigma}}\right)-w_1\right)+w_1+\pi_{i,1}$$
$$kl_{1,2}\left(w_2+(w_2)^{\frac{1}{\sigma}}\right)=-\frac{W}{K}\left(l_{1,1}\left(w_1+(w_1)^{\frac{1}{\sigma}}\right)-w_1\right)+w_2+\pi_{i,2}.$$

It follows that

$$k(w_1-w_2)=(w_1-w_2)+\pi_{i,1}-\pi_{i,2}$$

giving

$$\begin{aligned}
k&=1+\frac{\pi_{1,1}-\pi_{i,2}}{w_1-w_2}\\
&=1+\frac{\sum_j\left(f_1(L_{j,1})-w_1L_{j,1}\right)-\sum_j\left(f_2(L_{j,2})-w_2L_{j,2}\right)}{w_1-w_2}\\
&=1+\frac{\sum_i c_{i,1}-\sum_j(w_1L_{j,1})-\sum_i c_{i,2}+\sum_j(w_2L_{j,2})}{w_1-w_2}\\
&=1+\frac{\sum_i c_{i,1}-\sum_i\left(w_1(1-l_{i,1})\right)-\sum_i c_{i,2}+\sum_i\left(w_2(1-l_{i,2})\right)}{w_1-w_2}\\
&=1+\frac{(W+kK)(c_{1,1}+l_{1,1}-c_{1,2}-l_{1,2})-(w_1-w_2)n}{w_1-w_2}\\
&=1+\frac{(W+kK)(w_1-w_2)-(w_1-w_2)n}{w_1-w_2}\\
&=1+W+kK-n=1+(k-1)K
\end{aligned}$$

Conclude that $k=1$ which can happen only if the property owners' income is equal to those of workers.

3. Calibration

In this section, we numerically analyze a certain tax/subsidy policy designed to reduce the inefficiency discussed above. The particular method of achieving an increase in efficiency in this paper is redistribution of income among workers and capitalists. To this end, we need to take three steps: formally introduce the tax/subsidy scheme, specify a production technology in open form, and finally calibrate the parameters of preferences and technology.

3.1 Taxation Policy

Now let us define the redistribution policy which only consists of generalizing the budget set. In particular, let

$$\beta_i(p,w,q)=\left\{(c_i,l_i,z_i):\begin{array}{c}p_0c_{i,0}+w_0l_{i,0}+qz_i+t_{i,0}=w_0+\\ p_sc_{i,s}+w_sl_{i,s}+t_{i,s}=z_i+w_s+\\ for\ each\ s=1,2\end{array}\right. \quad (6)$$

where $t_i=(t_{i,0},t_{i,1},t_{i,2})$ is the vector of tax/subsidy that individual i pays/receives at each possible state of the world. If $t_{i,s}>0$ then individual i pays a tax in state s while she receives a subsidy otherwise. The budget balancedness condition for the government requires

$$\sum_{i=1}^n t_{i,s}=0 \text{ for all } s.$$

After writing $t=(t_1,\dots,t_n)$ the same condition becomes

$$\sum_{i=1}^n t_i=0.$$

Now we can define competitive equilibrium with incomplete markets and taxation.

Definition 1: CEI with Taxation is a vector $\langle (c_i,l_i,z_i)_{i=1}^n,(L_j)_{j=1}^n,q,(p_s,w_s)_{s=0,1,2}\rangle$ such that (c_i,l_i,z_i) solves (2) for each i with the budget constraint in (6), and L_j solves (1) for each j, and product, labor, and asset markets clear:

$$\sum_i c_{i,s}-\sum_j f_s(L_{j,s})=0$$
$$\sum_i l_{i,s}+\sum_j L_{j,s}-n=0$$
$$\sum_i z_{i,s}=0$$
$$\sum_{i=1}^n t_{i,s}=0$$

for each $s=0,1,2$ where the vector of taxation t is fixed.

In equilibrium, the utility of individual i is

$$U_i^*(t)$$

which is a function of the taxation policy t. Note that $U_i^*(0)$ corresponds to utility when there is no taxation, i.e. *laissez-faire*. That the laissez-faire equilibrium is Pareto-inefficient is proved in the previous section. Motivated by this observation, we will study taxation policies that satisfy $U_i^*(t)\geq U_i^*(0)$ for all i with strict inequality for some i. Hence, by definition, these policies induce a Pareto-improvement and reduce the inefficiency.

Since all individuals are either identical workers or identical capitalists, let us proceed with a representative worker, and a representative capitalist. The utility of the representative worker and capitalist are $U_W^*(t)$ and $U_K^*(t)$, respectively. In a similar vein, the tax that the representative worker and capitalist pay are t_W and t_K, respectively, implying the budget balancedness condition for the government is

$$Wt_W + Kt_K = 0$$

where W and K are the numbers of workers and capitalists, respectively.

Assume that the objective of the government is to increase the expected equilibrium utility of the workers as much as possible without harming the capitalists. In other words, the government solves

$$\max_{t_W, t_K} U_W^*(t) \quad (7)$$
$$\text{s.t.}$$
$$U_K^*(t) \geq U_K^*(0)$$
$$Wt_W + Kt_K = 0$$

The first constraint means that when there is taxation the expected utility of the capitalists do not fall short of their expected utility when there is no taxation. The second constraint is the budget balanced condition as discussed above.

3.2 Technology

Let us start with specifying the production technology in open form. Assume that output by firm j which employs L_j amount of labor at states s is

$$f_s(L_j) = \left(A_s^v + L_j^v\right)^{1/v}$$

where $A_s > 0$ is a state-dependent productivity parameter and $1/v$ is the elasticity of substitution between employment and the state dependent parameter. Write $A = (A_0, A_1, A_2)$ for the vector of all possible technological parameters.

The technology exhibits constant elasticity of substitution between the productivity parameter A_s and labor L_s. This constant elasticity of substitution is $1/(1-v)$. Therefore, v cannot be higher than 1. If $0 < v < 1$ then an increase in A_s reduces the marginal productivity of labor at state s. Otherwise, i.e. when $v < 0$, the marginal productivity of labor increases when A_s increases. Hence, we call an increase in A_s as a labor-saving technological progress, and capital saving if $0 < v < 1$.

3.3 Calibration

Now we can discuss the calibration of the parameters of the model. The vector of exogenous parameters of the model is

$$\xi = (v, A, \theta, \sigma, \mu).$$

The baseline values of these parameters are in the table below.

Table 1: Parameters' baseline values

v	$A = (A_0, A_1, A_2)$	$\theta = (\theta_0, \theta_1, \theta_2)$	σ	μ
-0,136	(70,84,70)	$\left(1, \frac{1}{2}, \frac{1}{2}\right)$	1,4	6,5

The mean of elasticity of substitution estimates by Antras (2004) is $1/(1-v) = 0.88$. Therefore, the calibration value is chosen as $v = -0{,}136$.

As for the state-dependent productivity vector, we set $A_0 = 70$ to ensure that the labor share in income is 2/3 when $t = 0$, i.e laissez-faire. $A_1 = 84$ implies that the potential increase in this technological parameter is 20% while $A_2 = 70$ means state 2 corresponds to no technological progress in the future. According to the discussion above, this is a labor saving technological progress since $v = -0{,}136 < 0$.

As for $\theta = (\theta_0, \theta_1, \theta_2)$ which gives the probability of each state, by definition, $\theta_0 = 1$. We set $\theta_1 = 1/2$ following the estimates of Frey and Osborne (2017) implying $\theta_2 = 1/2$. Finally, σ and μ are set to 0,85 and 3,65, respectively to ensure that Frisch elasticity of labor supply is 0,4 and average labor supply is 20% of labor endowment. See (Reichling and Whalen (2012)) for the estimates of Frisch elasticity of labor supply. According to the US Bureau of Statistics, the annual per capita working hour in the US is approximately 1800 hours implying

$$\frac{1800}{365 \times 24} = 0{,}2.$$

Table 2: Results of the calibration at baseline parameter values

Case	$t_W = (t_{W,0}, t_{W,1}, t_W$	$t_K = (t_{K,0}, t_{K,1}, t_K$	U_W^*	U_K^*
1	$(+,-,0)$	$(-,+,0)$	-98.3742	-81.382
2	$(-,0,+)$	$(+,0,-)$	-98.3742	-81.382
Laissez-faire	(0,0,0)	(0,0,0)	-98.3789	-81.382

3.4 Results

This section discusses the solution in t_W and t_K to the government's problem given in (7) when the parameters of the model are set to their baseline values in Table 1. Two separate cases are considered: $t_{W,1} = 0$ (which also implies $t_{K,1} = 0$) and $t_{W,2} = 0$ (which implies $t_{K,2} = 0$). As we shall soon see, these constraints are immaterial to the welfare of individuals.

In Case 1, workers are taxed in the initial state to be subsidized if labor saving technological progress takes place. In Case 2, workers are subsidized to be taxed in case labor saving technological progress does not occur. As can also be seen Table 2 above, utility in equilibrium is the same for both workers and capitalists in Case 1 and 2. This implies there is no impact of imposing one of the tax rates to zero on welfare. The increase in utility by taxation can be seen by comparing U_W^* in laissez-faire to that in Case1 (or, Case2).

The table below shows the equilibrium wages with and without government intervention. It is surprising that equilibrium wages are the same regardless of whether there is taxation or not. This implies that equilibrium level of employment and outputs are identical in Case 1 and 2 and laissez-faire. As a matter of fact, the difference between equilibrium with and without taxation stems from the difference in leisure between capitalists and workers.

Table 3: Equilibrium wages

Case	$w = (w_0, w_1, w_2)$
1	(0.018, 0.01, 0.18)

2	$(0.018, 0.01, 0.18)$
Laissez-faire	$(0.018, 0.01, 0.18)$

Table 4 clearly shows that the solution to the inefficiency of the laissez-faire equilibrium by taxation causes the workers to enjoy more leisure at the initial state, and the state in which there is labor saving technological progress, while capitalists enjoy more leisure at the future state without any technological progress.

Note that the values of taxes are derived by solving an optimization problem: maximizing workers' utility in equilibrium such that capitalists are not worse-off. But this outcome may or may not be Pareto-efficient. To see, the Pareto-efficiency properties of the taxation problem, let us seek the solution to

$$\max\ U_W(c_W, l_W)$$
$$\text{s.t.}$$
$$U_W(c_K, l_K) \geq U_K^*(0)$$
$$\langle (c_i, l_i, z_i)_{i=W,K}, (L_j)_{j=1}^{n} \rangle \ is\ feasible.$$

Again, surprisingly, the solutions to this Pareto-efficiency problem are identical to those of Case 1 and Case 2. Therefore, the taxation policy that we analyze in this study fully achieves Pareto-efficiency. This also explains why Case 1 and Case 2 induce identical outcomes. The reason is that they correspond to the unique solution of the Pareto-efficiency problem above.

Table 4: Equilibrium leisure

Case	$l_W = (l_{W,0}, l_{W,1}, l_{W,2})$	$l_K = (l_{K,0}, l_{K,1}, l_{K,2})$
1	$(0.7, 0.7, 0.7)$	$(1.65, 1.7, 1.57)$
2	$(0.7, 0.7, 0.7)$	$(1.65, 1.7, 1.57)$
Laissez -faire	$(0.698, 0.69, 0.71)$	$(1.66, 1.71, 1.56)$

3.5 Robustness

In this subsection, the numerical simulations are repeated by adding perturbation to the baseline parameter values. The most crucial parameters are v and σ. Recall that v gives the elasticity of substitution between labor and technology while σ corresponds to elasticity of substitution between leisure and consumption.

Table 5: Perturbing elasticity of substitution in technology

v	$t_W = (t_{W,0}, t_{W,1}, t_{W,2})$	$t_K = (t_{K,0}, t_{K,1}, t_{K,2})$
-0.12	(0.0006, -0.0002, 0)	(-0.005, 0.002, 0)
-0.11	(0.0003, -0.0001, 0)	(-0.003, 0.001, 0)
-0.1	(0.0001, -0.00007, 0)	(-0.0017, 0.0006, 0)
-0.09	(0.00009, -0.00003, 0)	(-0.0008, 0.0002, 0)
-0.08	(0.00003, -0.00001, 0)	(-0.0003, 0.0001, 0)

Let us start with v whose base value is -0,136. As can be seen in Table 5 when v increases and all other parameters remain fixed, the absolute value of taxes that restore efficient allocations in equilibrium get smaller. In other words, taxes that ensure efficiency are higher when inputs are complements in a stronger fashion. However, signs of t_W and t_K are persevered despite changes in v.

Table 6: Perturbing elasticity of substitution in utility

σ	$t_W = (t_{W,0}, t_{W,1}, t_{W,2})$	$t_K = (t_{K,0}, t_{K,1}, t_{K,2})$
0,83	(0.0077, -0.0033, 0)	(-0.007, 0.003, 0)
0,84	(0.0073, -0.0031, 0)	(-0.0065, 0.0028, 0)
0,85	(0.0069, -0.003, 0)	(-0.0062, 0.0027, 0)
0,86	(0.0065, -0.0028, 0)	(-0.0058, 0.0025, 0)
0,87	(0.0061, -0.0027, 0)	(-0.0055, 0.0024, 0)

Finally, we can focus on perturbing σ, which gives the elasticity of substitution between leisure and consumption, $\sigma/(\sigma-1)$. Recall that the baseline value for σ is 0.85. The results in Table 6 show that, as σ increases inducing lower elasticity of substitution, the absolute value of taxes decrease. In other words, complementarity between leisure and consumption causes taxes that restore efficient outcomes are smaller. Note that this relation between complementarity and taxation is the opposite of the relation that we see in case of perturbing v.

4. Conclusion

Motivated by the high possibility of widespread substitution of labor by robots and computers, this study asks "What are the policy implications of replacing humans with machines in the production process?" This question is typically asked in the context of equality of income. Yet our concern is not equity, but efficiency.

The paucity of an insurance against the adverse effects of a possible change in future technology due changes in relative prices ensures that the competitive equilibrium is inefficient. However, our numerical simulations show that redistribution of income can solve this problem. The results can be summarized as follows. Either workers should be taxed today to finance their subsidies in case of a labor saving technological progress in the future in order to cover their losses, or capitalists should be taxed today to finance their subsidies in case of no technological progress in the future, in order to cover their losses.

These results can be useful in guiding future economic policies of redistribution of income to prevent the negative impacts of uncertainty in technological change. Of course, the fact that the model is a stripped-down version of reality evokes the obvious need for further research in this area.

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