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A Time-Varying Parameter Vector Autoregression Model for Forecasting Emerging Market Exchange Rates

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Abstract

In this study, a vector autoregression (VAR) model with time-varying parameters (TVP) to predict the daily Indian rupee (INR)/US dollar (USD) exchange rates for the Indian economy is developed. The method is based on characterization of the TVP as an optimal control problem. The methodology is a blend of the flexible least squares and Kalman filter techniques. The out-of-sample forecasting performance of the TVP-VAR model is evaluated against the simple VAR and ARIMA models, by employing a cross-validation process and metrics such as mean absolute error, root mean square error, and directional accuracy. Out-of-sample results in terms of conventional forecast evaluation statistics and directional accuracy show TVP-VAR model consistently outperforms the simple VAR and ARIMA models.

Keywords: Stock Prices, Exchange Rates, Bivariate Causality, Forecasting

JEL Classification: C22, C52, C53, F31, G10

1. Introduction

Various significant structural transformations between 1960 and early 1970s led to the dramatic end of the Breton-Woods system of pegged exchange rates. Numerous efforts to bring back the fixed exchange rate system proved futile and by March 1973, the regime of floating currencies began. Collapse of Breton-Woods and rapid expansion of global trading markets has altered the dynamics of foreign exchange market dramatically. This market is considered the largest and most liquid of the financial markets, with an estimated $1 trillion traded every day.

So, in such an environment where exchange rate fluctuates, policymakers strive to understand the exchange rate movements and their implications on interest rates and inflation. The interest rate is set on the basis of an overall assessment of the inflation outlook. Moreover, exchange rate movements affect consumer price inflation through

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various means. Therefore, fluctuation in the rates is considered a source of economic disturbance and helps determine future inflation.

Multinational companies (MNCs) thrive on most of their sales coming from overseas. And for this, they have to dabble with many tasks such as decisions on hedging, short-term financing, short-term investment, capital budgeting, earning assessment, and long-term financing. This makes MNCs vulnerable to exchange rate movements. Thus understanding the exchange rate movements can help MNCs take better decisions. Large volume and value of currencies transacted around the world have also led to speculation in these markets, as huge benefits are netted from the differences in exchange rates.

Moreover, post Breton-Woods, many countries including India resorted to managed floating exchange rate system. In this system, the central bank checks the free movement of exchange rates and intervenes in the market, as a buyer or seller of currencies to influence adverse market conditions or exchange rate movements. What is more, it is needless to say that policymakers need to have a thorough knowledge of currency movements.

As exchange rates are considered an important input in various decision-making processes, numerous significant contributions (Frankel, 1976; Meese and Rogoff, 1983a, 1983b; Alexander and Thomas, 1987; Wolff, 1987; Baillie and McMahon, 1989; Mussa, 1990; Gallant et al.,1991; Meese and Rogoff, 1991; Frankel and Rose, 1995) have been made theoretically and empirically to characterize and understand exchange rates. However, results of most studies (Meese and Rogoff, 1983a; 1983b; Alexander and Thomas, 1987; Somanath, 1986; Boothe and Glassman, 1987; Wolff, 1987; Wolff, 1988) suggest that econometric forecasting models of exchange rates failed to outperform the random walk model, in forecasting variations in exchange rates. These results also point out that the exchange rate is an asset price and is determined efficiently in the foreign exchange market.

Results of few studies (Woo, 1985; Schinasi and Swami, 1989; Kuan and Liu, 1995; Brooks, 1997; Gencay, 1999; Kumar and Thenmozhi, 2004, 2005; Hongxing et al., 2007; An-Pin Chen et al., 2008) contradict the earlier findings and show that their models outperform the naïve random walk, for certain time periods and currencies. These studies conclude that the presence of nonlinearity, volatility, chaos, nonstationarity, etc in the exchange rates was not handled properly in previous empirical models. This resulted in the random walk scoring over other models.

Most studies (Brooks, 1997; Panda and Narsimhan, 2003; Corte et al., 2007; Rime et al., 2007, An-Pin Chen et al., 2008 etc.) on exchange rate forecasting have either used macroeconomic variables such as inflation rates, money supplies, interest rates, trade balance, and crude oil price; or technical indicators such as past returns of exchange rates, short- and long-term moving averages etc as an input variables to the forecasting models.

Moreover, studies (Lisi and Schiavo, 1999; Qi and Wu, 2003; Kumar and Thenmozhi, 2003, 2004 and 2005; Chen and Leung, 2004; Cao et al., 2005; Hongxing et al., 2007; An-Pin Chen et al., 2008; Carriero et al., 2009) in the domain of exchange rate forecasting have used a wide range of linear and nonlinear models such as autoregressive models, ARIMA, linear regression, VAR, ARCH/GARCH, artificial neural networks

(ANN), and support vector machines (SVM). Earlier studies also used a wide range of statistical metrics such as root mean square error (RMSE), mean absolute error (MAE), mean absolute percentage error (MAPE), and Theil's coefficient—to evaluate the performance of forecast.

During the last decade, the use of various nonlinear models such as ANN and SVM in forecasting various financial time series has considerably been increased. However, having a forecasting model which is consistent with economic theory and also forecast well is very appealing. So, in this study, we use the VAR time series approach to forecast the daily exchange rates of INR vs. USD. We select INR/USD rates because India is the world's sixteenth largest foreign market, in terms of daily turnover ($34 billion in 2007). India's contribution to global foreign market turnover has grown to 0.9% in 2008, a three-fold jump from just 0.3% in 2004. It also recorded the second-highest growth in the daily average foreign exchange market turnover after China. In 2006–07, India's annual gross foreign exchange market turnover grew to $6.5 trillion from $1.4 trillion, six years earlier. In August and October 2008, the Security and Exchange Board of India (SEBI) permitted National Stock Exchange (NSE), Bombay Stock Exchange (BSE) and Multi Commodity Exchange of India (MCX) to set up a currency derivative segment as well, where only currency futures (INR/USD) contracts are traded. Total turnover in this segment increased to $19.52 billion in March 2009 from $3.38 billion in October 2008. Open interest in the segment also grew 96% to 4, 51,819 contracts in March 2009 from 2, 30,257 contracts in October 2008.

Moreover, India has attracted unprecedented foreign investment (touching a phenomenal $10 billion) and is poised to become a major hub in the Asian economy. With the growing interest and research on emerging markets, India remains in the focus due to its rapid growth and potential investor opportunities. Moreover, volatility in emerging markets seems to be higher than in the developed markets—often making prediction difficult. This background makes the study more worthwhile: whether the dynamic linkages between INR/USD and stock market indices in India can be deployed to build a superior and accurate forecasting model.

This specific study improves upon the existing studies in several ways. To develop a successful model, it is necessary to consider the structural change in data. For this, 'regime-switching' and 'time varying parameter' (TVP) are two popularly used techniques; The TVP approach is used in the study given. This approach is beneficial in the sense that it makes use of all available data points and retains the long-term relationship inherent in the old data points (Hongxing et al., 2007). The study characterizes the TVP of VAR model based on the optimal control theory as proposed by Rao (2000). This methodology is a blend of flexible least squares and Kalman filter technique. Rao (2000) estimated TVP for linear regression models without an intercept; however, this study extends Rao's approach. Our method updates the coefficient of the VAR model and its covariance matrices in each time unit. Thus, the model can also be called a Bayesian VAR model. There is also a comparison between the results of the time-varying VAR model given with those of a linear VAR and ARIMA models.

Earlier literature used macroeconomic variables, technical indicators, etc to develop forecasting models; but not stock prices data. Several recent studies (Vygodina, 2006; Pan et al., 2007; Ai-Yee Ooi, 2009; Aydemir and Demirhan, 2009 etc.) have reported causality from stock prices to exchange rates. Their results support the goods market or portfolio approach. Many firms in India that are on the stock index, NSE's S&P CNX Nifty (thereafter, CNX) have American Depository Receipts (ADRs) or General Depository Receipts (GDRs) that are traded on the NYSE, NASDAQ or on non-American exchanges. Over the years, INR has gradually moved towards full convertibility. The two-way fungibility of ADRs/GDRs allowed by India's central bank, Reserve Bank of India (RBI) has also possibly enhanced the linkages between the stock and foreign exchange markets in India. So, the dynamic relations between a stock index and exchange rates are examined, using linear granger causality tests. In addition, there is also usage of unit root and cointegration tests to analyze the long-run equilibrium between the two variables. The study exploits the dynamic linkage between stock price and exchange rate, and uses the granger causality test results to select important inputs for forecasting INR/USD.

Most previous studies arbitrarily split the available data into training (in-sample) and test (out-of-sample) set for model construction and validation, respectively. Performance of the forecasting model is likely to vary with the variation in time. So, consistency and robustness of the several competing models on various test data should also be evaluated. So in our study, we employ a cross-validation methodology (in-sample and out-of-sample data sets) to investigate the performance of various forecasting models.

Besides, earlier studies measured the degree of accuracy and acceptability of forecasting models, by the estimate's deviations from the observed values; they have not considered turning-point forecast capability using sign and direction test. In this study, there is a rigorous evaluation of the performance of forecasting models using MAE, RMSE, and directional accuracy. Directional accuracy measures the degree to which the forecast correctly predicts the direction of change in the actual INR/USD exchange rate returns.

It is expected that the outcome of the study will help enhance the existing literature, and offer some meaningful insights for policy makers, MNCs, traders, and individuals. The paper is set out as follows. In Section 2, daily exchange rates and the concept of unit root tests, cointegration tests, linear granger causality, VAR, TVP-VAR framework and ARIMA models are described. In Section 3, the empirical results are presented. Finally, Section 4 concludes with some discussion on future research.

2. Data and Methodology

We examined the daily closing price of CNX and INR/USD, obtained from NSE and RBI Web sites, respectively. The time period used is from January 4, 1999 to August 31, 2009. The original series was transformed into a continuously compounded rate of return, computed as the first difference of the natural logarithm of CNX and INR/USD.

2.1 Unit Root Tests

To test the unit roots (stationarity) in CNX and INR/USD exchange rates, Augmented Dickey and Fuller (ADF) and Kwiatkowski, Philips, Schmidt and Shin (KPSS) tests are used. An ADF is a test for a unit root in a time series sample. KPSS tests are used for testing a null hypothesis that an observable time series is stationary around a deterministic trend. KPSS type tests are intended to complement unit root tests, such as the Dickey–Fuller tests. Table 1, contains the results.

2.2 Engle and Granger Cointegration Test

To investigate the existence of long-run relationship between two variables (CNX and INR/USD), the methodology suggested by Engle and Granger (1987) is employed. We prefer to use this method over Johansen Cointegration test, as Engle and Granger test is simple and helps in having at most one cointegrating vector (because we are examining two variables).

In the first step, we examine the order of integration of each variable. Cointegration between CNX and INR/USD requires that both series should be non-stationary of the same order of integration. In the second step, we run the following cointegration regression:

$$\ln S_t = \gamma_0 + \gamma_1 \ln ER_t + \varepsilon_t \tag{1}$$

Where ln S_t and ln ER_t are logarithms of CNX and INR/USD, respectively.

The third step is to obtain the error terms and run the ADF and KPSS tests on them in order to examine whether error terms have unit root and are stationary or not. If the error series is stationary, then the null hypothesis of no-cointegrating vectors is rejected. Table 2, lists the results of Engle and Granger cointegration test.

2.3 Granger Causality Test

In literature, various tests of Granger causality have been proposed and used. These tests are mainly based on the context of VAR models. So we employ the VAR framework to examine the presence of linear Granger causality between CNX and INR/USD. Let V_t denote the vector of endogenous variables and *p*, the number of lags. Then the VAR model can be represented as:

$$V_t = \sum_{i=1}^{p} \Psi_i V_{t-s} + \varepsilon_t \tag{2}$$

Where $V_t=(V_{1t,\ldots,}V_{pt})$, the p x 1 vector of endogenous variables, Ψ_i the p x p coefficient matrices, and ε_t is a zero-mean vector of white-noise processes.

A VAR model including CNX returns denoted as X and INR/USD returns as Y can be expressed as:

$$X_t = \alpha(p)X_t + \beta(p)Y_t + \varepsilon_{xy,t} \qquad t = 1,2,....,N \tag{3}$$

$$Y_t = \chi(p)X_t + \delta(p)Y_t + \varepsilon_{yx,t} \qquad t = 1,2,....,N \tag{4}$$

where $\alpha(p)$, $\beta(p)$, $\chi(p)$ and $\delta(p)$ are all polynomial in the lag operator with all roots outside the unit circle

The error terms are identically and independently distributed (i.i.d) with zero-mean and constant variance.

If cointegration exists between CNX and INR/USD series, then the Granger representation theorem states that there is a corresponding error correction model. This model for CNX and INR/USD series can be represented as:

$$X_t = \alpha(p)X_t + \beta(p)Y_t + \phi Z_{t-1} + \varepsilon_{xy,t} \qquad t = 1,2,....,N \tag{5}$$

$$Y_t = \chi(p)X_t + \delta(p)Y_t + \varphi Z_{t-1} + \varepsilon_{yx,t} \qquad t = 1,2,....,N \tag{6}$$

Where $Z = \ln S_t - \gamma_0 - \gamma_1 \ln ER_t$ are the residuals from the cointegration regression of the log levels and $\Delta\ln S_t$ and $\Delta\ln ER_t$ is the first log difference of CNX and INR/USD, respectively (or simple exchange rate and CNX returns). Optimal lag length is selected based on the Akaike Information Criteria (AIC).

Within the context of this VAR/VECM model, linear Granger causality restrictions can be defined as follows. If the null hypothesis that β's jointly equal zero is rejected, it is argued that INR/USD returns (Y) Granger-cause CNX returns (X). Similarly, if the null hypothesis that χ's jointly equal zero is rejected, CNX returns (X) Granger-cause exchange rate returns (Y). If both the null hypotheses are rejected, a bi-directional Granger causality, or a feedback relation, is said to exist between variables. Different test statistics have been proposed to test for linear Granger causality restrictions. To test for strict Granger causality for pairs of (X,Y) in this linear framework, Chi-Square statistics is used to determine whether lagged value of one time series has significant linear predictive power for the current value of another series. Table 3 lists the results.

2.4 ARIMA Model

Popularly known as Box-Jenkins (BJ) methodology, but technically known as Autoregressive Integrated Moving Average (ARIMA) model, it is of the following form:

$$Y_t = a_0 + \sum_{i=1}^{p} \alpha_i Y_{t-i} + \sum_{i=0}^{q} \beta_i e_{t-i} + \xi_t \tag{7}$$

Where Y_t is INR/USD returns, ξ_t is an uncorrelated random error term with zero mean and constant variance, and a_0 is a constant term.

The correlogram, simply a plot of Autocorrelation Functions (ACFs) and Partial Autocorrelation Functions (PACFs) against the lag length, is used in identifying the significant ACFs and PACFs. Lags of ACFs and PACFs, whose probability is less than 5%, are significant and are identified. Possible models are developed from these plots for CNX returns series. The best model for forecasting is picked by considering information criteria such as AIC and Schwarz Bayesian Information Criterion (SBIC).

2.5 The VAR Methodology

In this study, a VAR model is employed to forecast the INR/USD returns. Given V_t the vector of variables (CNX (X_t) and INR/USD (Y_t) returns), the classical first-order bi-variate VAR model can be represented as:

$$V_t = \Psi V_{t-1} + \varepsilon_t \qquad t = 1, \ldots N \tag{8}$$

where $V_t = (Y_t, X_t)'$, Ψ is a 2 x 2 coefficient matrix and ε_t is a zero-mean vector of white-noise processes, with positive definite contemporaneous covariance matrix Σ and zero covariance matrices at all other lags.

To estimate the parameters, our study uses the least squares method. Optimal lag length is selected using information criteria (AIC and SBIC).

Optimal one-period ahead forecast of V_t using the VAR model can be obtained using the below equation:

$$V_{t+1,t} = \Psi V_t \tag{9}$$

2.6 VAR Model with TVP

Traditional time series model with time-invariant coefficients cannot be used to examine the relationship among economic variables, as they alter with changes in economic policies. So time variation of these parameters should go hand in hand with policy changes.

Thus, several estimation methods (switching regression model (Quandt, 1958; Goldfled and Quandt, 1973), pure random coefficient model (Rao, 1965), adaptive and varying parameters model (Cooley and Prescott, 1973a; 1973b), Kalman filter model (Athan, 1974), flexible least squares (Kalaba and Tesfatsion, 1988), recursive model

(Rao, 1991), optimal control model (Rao and Nachane, 1988) have been proposed in the literature to deal with the problem is estimating TVP models.

Rao (2000), proposed a methodology to estimate the TVP of a linear regression model without an intercept term. The methodology is based on the characterization of the TVP problem as a near-neighborhood search problem, with an explicit allowance for welfare loss considerations. This leads to an updating algorithm, capable of predicting the optimal values of TVP as well as their covariance matrices. This methodology is a blend of the flexible least squares and Kalman filter techniques.

In the present study, there is an attempt to extend Rao (2000), to estimate TVP of VAR model. As the method updates the VAR coefficients and their covariance matrices for every time period, it could be considered as a Bayesian method. As discussed in the earlier section, the classical linear first-order bivariate VAR model is represented as:

$$V_t = \Psi V_{t-1} + \varepsilon_t, \qquad t = 1,....N \tag{10}$$

In the optimal control theory, the TVP of VAR model using near-neighborhood search method can be represented as:

$$V_t^* = \Psi_t^* V_{t-1} \qquad t = 1,....N \tag{11}$$

A control problem comprises a cost function, which indeed is a function of state and control variables. The linear quadratic cost functional, which has to be minimized for the TVP VAR model, can be stated as:

$$W = \frac{1}{2}\sum_{t=1}^{N} Q_t (V_t^* - V_t)^2 + \frac{1}{2}\sum_{t=1}^{N} (\Psi_t^* - \Psi_{t-1}^*)' R_t (\Psi_t^* - \Psi_{t-1}^*) \tag{12}$$

Where V_t, V_t^* are states, Ψ_t^* is control, and t is the time. In the finite-horizon case, the matrices Q_t and R_t are positive semi-definite and positive definite, respectively. The solution of the optimal control problem may not be unique. Most often, the solutions of such problems is locally minimizing. However, the main advantage of the cost function is that the constraints Ψ_t^* lie within the neighborhood of Ψ_{t-1}^* (Rao, 2000).

Equation 12, can be solved using the method of Lagrange multipliers. It is converted into an equivalent cost function, using certain unspecified parameters known as Lagrange multipliers (λ_t). The new cost function called as Lagrange function can be defined as:

$$J = \frac{1}{2}\sum_{t=1}^{N} Q_t (V_t^* - V_t)^2 + \frac{1}{2}\sum_{t=1}^{N} (\Psi_t^* - \Psi_{t-1}^*)' R_t (\Psi_t^* - \Psi_{t-1}^*) - \sum_{t=1}^{N} \lambda_t (V_t^* - \Psi_t^* V_{t-1}) \tag{13}$$

To obtain the solution of the new cost function, we take the partial derivative of J with respect to three variables V_t, Ψ_t^* and λ_t; and set the partial derivative to zero. We get:

$$\frac{\delta J}{\delta V_t^*} = Q_t(V_t^* - V_t) - \lambda_t = 0 \qquad (t = 1,\ldots..,N) \tag{14}$$

$$\frac{\delta J}{\delta \Psi_t^*} = R_t(\Psi_t^* - \Psi_{t-1}^*) + V_{t-1}\lambda_t = 0 \qquad (t = 1,\ldots..,N) \tag{15}$$

$$\frac{\delta J}{\delta \lambda_t} = -(V_t^* - \Psi_t^* V_{t-1}) = 0 \qquad (t = 1,\ldots..,N) \tag{16}$$

Solving Equation 14 for λ_t gives:

$$\lambda_t = Q_t V_t^* - Q_t V_t \tag{17}$$

Solving for V_t^* using Equation 16, we get:

$$V_t^* = \Psi_t^* V_{t-1} \tag{18}$$

Substituting the value of V_t^* in Equation 17 yields:

$$\lambda_t = Q_t \Psi_t^* V_{t-1} - Q_t V_t \tag{19}$$

Substituting Equation 19, in Equation 15, results in:

$$\Psi_t^* = (R_t + V_{t-1}Q_t V_{t-1})^{-1} R_t \Psi_{t-1}^* + (R_t + V_{t-1}Q_t V_{t-1})^{-1} V_{t-1}Q_t V_t \tag{20}$$

At this point, we have all the variables in terms of Ψ_t^*
In Equation 20, if the variable $Q_t = 0$, then it becomes:

$$\Psi_t^* = \Psi_{t-1}^* \tag{21}$$

Equation 21, suggests that the coefficient of the VAR model is time-invariant.
However, in Equation 15, if we set $R_t = 0$, then it becomes:

$$V_{t-1}\lambda_t = 0 \tag{22}$$

From equation 19 and 22, we get:

$$V_{t-1}Q_t V_{t-1}\Psi_t^* = V_{t-1}Q_t V_t \tag{23}$$

Equation 23, results in $V_{t-1}\Psi_t^* = V_t$. This suggests that the dependent variables of VAR model can be tracked. Moreover, the adaptive nature of control system is evident, if Equation 20 is transformed. To do this, let us assume that:

$$K_t = (R_t + V_{t-1}Q_tV_{t-1})^{-1}V_{t-1}Q \tag{24}$$

Using Equation 24, Equation 20, can be rewritten as:

$$(R_t + V_{t-1}Q_tV_{t-1})^{-1}[(R_t + V_{t-1}Q_tV_{t-1}) - V_{t-1}Q_tV_{t-1}] = I - K_tV_{t-1} \tag{25}$$

Substituting Equations 24 and 25, in Equation 20, we get:

$$\Psi_t^* = \Psi_{t-1}^* + K_t(V_t - \Psi_{t-1}^*V_{t-1}) \tag{26}$$

Where K_t is the correction factor

Equation 26, can be used to compute the Ψ_t^* from its previous estimate. Subsequently, Ψ_t^* can be used to compute the predicted state of V_t^*.

The above three methodologies have been used to forecast the daily returns of INR/USD.

2.7 Estimation and Prediction

Examining the robustness of the forecast models is an interesting topic as well as a meaningful trial. This helps access the forecast performance vis-à-vis different sample data sets. To investigate the performance of the competiting forecasting models, four validation sets are used. In the first set, daily data of CNX and INR/USD from January 4, 1999–December 31, 2006 is used. The data is divided into an estimation period (in-sample data) from January 4, 1999–December 31, 2005, and a forecast period (out-of-sample data), from January 1, 2006–December 31, 2006. In the second validation set, daily data is considered from January 4, 1999–December 31, 2007. There are estimations conducted over period from January 4, 1999–December 31, 2006 and data from January 1, 2007–December 31, 2007 is reserved for the forecasting exercise. The third validation set covers a daily period from January 4, 1999 – December 31, 2008. The data is divided into an estimation sample from January 4, 1999–December 31, 2007, and a forecast sample from January 1, 2008– December 31, 2008. In the last validation set, there is daily data from January 4, 1999–August 31, 2009. The data is divided into two periods: January 4, 1999–December 31, 2008, for model estimation and is classified as in-sample and a period from January 1, 2009–August 31, 2009 is reserved for out-of-sample forecasting and evaluation.

3. Results

3.1 Unit Root Test

Results of ADF and KPSS tests for the two time series: CNX and INR/USD returns and for the entire time period are shown in Table 1.

Table 1: Unit Root Test

Variable	ADF Test t-statistics	KPSS Test t-statistics
ln S_t (Log level)	-0.6553	5.3562
Δ lnS_t (First Diff)	-36.7465	0.0890
ln ER_t (Log Level)	-1.3609	0.7149
Δ lnER_t (First Diff)	-52.5394	0.1765

Source: Author estimation

Note: The critical values for ADF and KPSS test at 1 % level are -3.4327 and 0.739 respectively.

These results indicate that the log level of CNX and exchange rates series have a unit root. However, ADF and KPSS tests test on the first log order difference for the two series: Δln S_t and Δln ER_t; and this confirm the stationarity of the two series.

3.2 Engle and Granger Cointegration Test

After testing for the unit root in the two series, the two-step Engle and Granger cointegration test was applied to examine whether the logarithms of INR/USD exchange rate and CNX are cointegrated. Table 2, reports the results of the cointegration regression.

Table 2: Engle and Granger Cointegration Test

Cointegrating Regression			
Coefficient	**Coefficient Value**	**t-statistic**	**Probability**
γ_o	25.3729	40.3541	0.0000
γ_1	-4.6699	-28.2934	0.0000
Unit Root Test of Cointegrating Errors			
ADF Test		**KPSS Test**	
t-statistics	**Critical Value (1%)**	**t-statistics**	**Critical Value (1%)**
-0.5415	-3.4327	5.4933	0.739

Source: Author estimation

To find out whether the variables are actually cointegrated, the cointegration error terms are tested for stationarity. ADF and KPSS test results clearly indicate that the error terms are nonstationary. So, the results suggest that there is no cointegrating vector and eventually no long-run relationship between exchange rate and stock indices for India. Hence, an error correction term need not be included in the Granger causality test equations. Findings of Engle and Granger Cointegration tests are consistent with the findings of previous studies for developed markets such as US, UK, and Japan as well as for Asian markets like India, Malaysia, and Pakistan.

3.4 Linear Granger Causality Test

The dynamic (causal) relationship between CNX and INR/USD returns is investigated using bi-variate VAR framework, without the error correction term. The appropriate lag length for VAR models is selected using SBIC. Granger causality test results are in Table 3.

Panel A of Table 3, reports the results of linear Granger causal test, while panel B reports the Granger causality results between volatility-filtered CNX and INR/USD returns.

Table 3: Linear Granger Causality Test

Null Hypothesis	Chi-Sq-Statistics	P-Value
Nifty Returns does not Granger cause INR/USD	8.2422	0.0162**
INR/USD does not Granger cause Nifty Returns	9.6352	0.0081*

Source: Author estimation

* Represent the relationship being significant at 1 %

** Represent the relationship being significant at 5 %

The optimal lag length is 2 which are selected based on the SBIC criteria.

The results reported in Panel A, indicate that both the null hypotheses ‘Nifty Returns does not Granger-cause INR/USD returns’ and ‘INR/USD returns does not Granger-cause Nifty returns’ are rejected. Chi-Square statistics are significant and provide strong evidence for the argument that there is a Bi-directional linear Granger causality between CNX and INR/USD returns.

In general, the results suggest that exchange rates do help explain changes in the stock index and vice versa. So, the results of our study do not support the ‘Efficient Market Hypothesis’ for the Indian market. Moreover, the findings strongly support the portfolio approach on the relationship between exchange rates and stock prices. Thus, we could use stock price as an initial attribute to forecast exchange rates and vice versa.

3.5 ARIMA Model

Plots of ACFs and PACFs are used to identify the significant lag length. Various orders of ARIMA models are developed using ACF and PACF plots. Information criteria (AIC and SIC) help identify the best forecasting model (results available upon request). After considering all possible models and looking at AIC and value of each model, it was decided that ARIMA (2,1,1) is best model for forecasting daily returns of INR/USD series for the first validation set (January 1, 1999–December 31, 2006). Moreover, for the subsequent validation data sets, ARIMA (1,1,2) is the best. Further diagnostic tests are performed to check the model's adequacy.

We use a popular diagnostic test known as Breusch-Godfrey LM test to examine the presence of serial correlation in the residuals of the developed ARIMA model. This test helps examine the relationship between residuals and several of its lagged values at the same time. The null hypothesis is that 'there is no serial correlation'. If the predictability value is greater than 5%, then we accept the hypothesis (at 95% confidence levels); so there is no serial correlation in the series. The LM test for serial correlation of residuals suggests that the ARIMA (2,1,1) and ARIMA(1,1,2) models capture the entire serial correlation; and the residuals do not exhibit any serial correlation (results available upon request). It suggests that the residuals, estimated by the two ARIMA models, are purely random. So, another ARIMA model may not be searched (Gujrati, 1995).

3.6 VAR Model

This model generally uses equal lag length for all its variables. One of its drawbacks is that many parameters need to be estimated, some of which may be insignificant. This problem of over-parameterization, resulting in multicollinearity and a loss of degrees of freedom, leads to inefficient estimates and possibly large out-of-sample forecasting errors (Litterman, 1986; Spencer, 1993). One solution, often adopted, is simply to exclude the insignificant lags based on statistical tests. Another approach is to use a near VAR, which specifies an unequal number of lags for the different equations.

In our study, while examining the causality test in the VAR framework, we selected two lags of CNX and INR/USD, based on SBIC criteria. However, when the parameters in VAR model of Equation 3 are estimated, it was found that the second lag of CNX and INR/USD seems to be insignificant. So we excluded these lags from the VAR model and re-estimated the model, using ordinary least squares criteria. So the forecast is done using a bi-variate first-order VAR model.

3.7 TVP-VAR Model

This model has been used to forecast the daily returns of INR/USD. First, an initial coefficient of TVP-VAR model is chosen that minimizes the welfare loss function. Estimates of the simple VAR model based on the in-sample data are used as an initial coefficient of the model for every cross-validation data set. The near-neighborhood approach using grid search is then employed on the initial coefficient to identify the coefficient values that further minimizes the loss function. This new coefficient as identified by the grid search is used to estimate the consecutive sequence of the coefficient and the corresponding predictions of the INR/USD returns, for each validation set of the model.

Forecasting performance of various models and for the four out-of-sample periods are summarized in Table 4.

Table 4: Prediction Accuracy

Model	**First Validation Test Set (January 1, 2006 –December 31, 2006)**		
	Performance Metrics		
	MAE	**RMSE**	**Directional Accuracy**
TVP-VAR	0.002065	0.002982	54.48%
VAR	0.002074	0.002986	53.13%
ARIMA	0.002081	0.002987	52.30%
Second Validation Test Set (January 1, 2007–December 31, 2007)			
Model	**Performance Metrics**		
	MAE	**RMSE**	**Directional Accuracy**
TVP-VAR	0.002550	0.003856	59.91%
VAR	0.002552	0.003861	56.61%
ARIMA	0.002555	0.003864	54.95%
Third Validation Test Set (January 1, 2008–December 31, 2008)			
Model	**Performance Metrics**		
	MAE	**RMSE**	**Directional Accuracy**
TVP-VAR	0.004687	0.006841	56.06%
VAR	0.004705	0.006885	53.13%
ARIMA	0.004759	0.006897	49.70%
Fourth Validation Test Set (January 1, 2009–August 31, 2009)			
Model	**Performance Metrics**		
	MAE	**RMSE**	**Directional Accuracy**
TVP-VAR	0.004725	0.006466	56.96%
VAR	0.004772	0.006493	55.69%
ARIMA	0.004872	0.006673	48.10%

Source: Author estimation

The results display the out-of-sample results of the various forecasting models. For the INR/USD returns, we find that the TVP-VAR model yields better forecast (smaller RMSE and MAE) than the simple VAR and ARIMA models, for all the validation sets.

Moreover, between VAR and ARIMA model, the VAR forecasts on the four out-of-sample data have smaller RMSE and MAE.

The TVP-VAR model also exhibits good market-timing ability as indicated by the results of directional accuracy. The directional accuracy of the model is 54–60% over the four test samples. This means the forecasts are comparatively better than the chances in tossing a coin. Compared to the ARIMA models, the simple VAR forecast have higher directional accuracy values.

As for the forecasting stability, two observations can be made from Table 4. First, the time series models (TVP-VAR) are robust across the cross-validation test and the results seem to be more stable. Second, no matter what method is used, there are no consistent patterns in MAE and RMSE across all out-of-sample periods. There is a difference in the values of various performance measures such as RMSE and MAE of TVP-VAR, VAR and ARIMA models for all out-of-sample periods. This result is expected since the structure of the exchange rate time series varies from one time period to the other. If in-sample and out-of-sample date generally increase or decrease or vice versa, then it is clear that no method can predict well particularly in the short run—leading to large variations in prediction. So, it may be concluded that the predictive accuracy of all models changes across time, for different forecasting horizons.

Overall, results suggest that the TVP-VAR model contains added information for INR/USD and strongly outperforms the other two models. They are consistent with our expectations that allowing TVP using optimal control theory enhances the model's forecasting performance.

4. Conclusion

A Bayesian VAR model was developed based on the optimal control theory, which updates the coefficient and their covariance matrices in each time period. Our TVP-VAR model helps forecast the daily returns of INR/USD. The results of the TVP-VAR model are compared with the simple VAR and a linear ARIMA model. A cross-validation scheme is employed to examine the robustness of the three models with regard to sampling variation in time series. Out-of-sample performances of the three models were evaluated along performance metrics like MAE, RMSE and Directional Accuracy. Results from the study indicate that the TVP-VAR model achieves high rate of accuracy, in terms of MAE, RMSE and Directional Accuracy for the four validation sets. The results in general supports the study of Carriero et al. (2009), Sarantis (2006), Kumar and Thenmozhi (2003, 2004 and 2005), and Chen and Leung (2004) etc. The forecast gains are due primarily to the time-variation of coefficients. Moreover, the results also suggest that, informational content of indicators like stock index can be exploited to improve the exchange rate forecasts. Thus, the results reject the efficient market hypothesis and lend support to the technical analysis.

The findings of the study would be of great interest to traders, MNC's, regulators and others. The better forecasting or understanding of the movements of exchange rate for the developing economy of Asia would help traders to devise more effective business and trading strategies and a proper decision on asset allocation. Moreover, based on the forecast, they can also take precautionary measure to reduce potential currency risk.

Corporate and MNC's can effectively use such models for their foreign exchange risk management plan/policy/programme. Such models would help them to reduce the volatility in profits after tax, cash flows, and to reduce the cost of capital and thus increase the value of the firm on one side of the pole and to reduce the risks faced by the management on the other side of the pole.

The TVP-VAR model would also help policy makers in India to intervene successfully and at the right time in the market in order to prevent overshooting and decisively break the momentum in currency movement. Thus, the policy makers can conduct a suitable monetary policy which will in turn achieve its desired objectives of price stability and higher economic activity. Moreover, the dynamic bi-directional causal relationship between exchange rates and stock index also suggests that, the SEBI and the central bank i.e. RBI in India should be very careful in conducting exchange rate policies or capital market polices as it may impact on the development of the financial markets.

The study can be extended by taking into account the set of potential macroeconomic input variables such as interest rates, consumer price index and industrial production, as well as technical indicators. Moreover, various trading strategies can be used to examine if trading profits can be obtained from the forecasting model used in the study. The other logical extension could be to combine the TVP-VAR model with some nonlinear models used in the financial time series literature.

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