Modeling Labor Supply through Duality and the Slutsky Equation

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Abstract

In the present paper an analysis of the neo-classical optimization model with linear constraints is proposed. By introducing the dual problem it is shown that the solution to the maximization problem is also a solution to the minimization problem. The purely theoretical model proposes a universal equation, similar to the Slutsky equation as derived in the consumption theory. Another application is needed, different from the standard applications of the model found in economic literature. This application is based on the study of the change in optimality caused by the taxes on labor. The application focuses on how they impact the optimal decision in the choice between leisure and labor through the application of the classification derived on the basis of the Slutsky equation.

Keywords: labor optimization, duality, the Slutsky equation, tax rates

JEL classification: C61, C62, D11

1. Introduction

The model in the present paper elaborates the ideas as proposed by Ivanov (2005) and it differs from the traditional application of the optimization method, which solves for the maximum value of an n-argument function under a given constraint. The dual problem has been recently used by Menez and Wang (2005), who analyze the income and substitution effect under an increase in wage risk and uncertainty. Sedaghat (1996) provides a version of the Slutsky equation in a dynamic consumer's account model. In this paper, we propose an optimization model of labor supply by introducing the so called 'dual' problem and we find solutions for maximum and minimum. Aronsson (2004), Jones (1993), Sorensen (1999), and Werning (2007) treat the problem of optimal taxation and decision making in defining fiscal policies. Similar analysis are proposed by Bassetto

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(1999) and Chari (1994; 1999) who formulate the idea of optimality in taxation and its impact on business cycle management and in environments with heterogeneous agents.

There are few studies on tax avoidance and its effects on labor supply and general welfare like for example the ones we find in Agell (2004), Gruber (2002), Hausman (1983), and Kopczuk (2005). In our paper, by applying the Slutsky equation we propose a general classification of variables, analogous to the classification of goods based on income and price elasticity and we further use it in the labor supply model to interpret the influence of taxes on the changes in the choice between labor and leisure.

The paper is organized as follows: in Section 2 the purely theoretical optimization model is presented. By solving the maximization problem and its dual, a universal equation is derived based on the Slutsky equation in the consumption theory and an analogous classification of the variables which are arguments of the objective function is proposed. In Section 3 there are some comments on the representations of the model in labor supply decision-making. Section 4 includes an analysis taxes and their impact on the labor supply process. Section 5 summarizes the results and concludes.

2. The Model

We consider the function $\varphi(x)=\varphi(x_1, x_2, ..., x_n)$, defined in a convex and compact set $x \in X \subset R^n$, which is *continuous, monotonic, twice differentiable, quasi-concave and homogeneous of degree one* and this set is also characterized by local non-satiation.

For the purposes of this analysis we will consider the function $\varphi(x)$ as an objective function which we want to maximize under a certain linear constraint. In the n-dimensional case the model takes the following form:

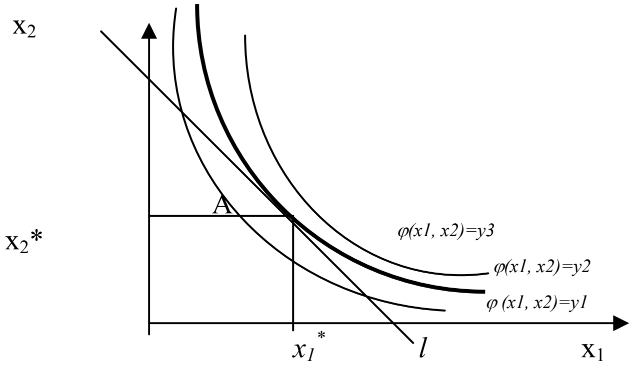
$$\max_{x \geq 0} \varphi(x)$$
$$\text{s.t. } \langle a, x \rangle \leq b$$

where $x = (x_1, x_2, ..., x_n)$ is a vector of the arguments of the objective function, $a = (a_1, a_2, ..., a_n)$ is a vector of parameters, which are positive numbers and influence the constraint. The scalar $b \geq 0$ determines the value of the constraint.

We will assume that if the function $\varphi(x)$ is continuous, then $\varphi(x) \leq \varphi(x^*)$ for all $x \in X$, the constraint $\langle a, x \rangle \leq b$ belongs to a full and compact set and the vector a>0. Then, the vector x^* is an optimal vector consisting of the arguments of the function $\varphi(x)$. We will also assume that the vector x^* is a global maximum and is also a solution to the problem.

For the geometrical representation we shall discuss the two-dimensional case (Fig. 1).

Figure 1: Optimization model



If we assume that the function $\varphi(x_1, x_2) = y$, where y is some number, for different values of x_1 and x_2 the objective function has one and the same value for y. On Figure 1 this function is presented by a family of curves, which correspond to changes in the value of y. These curves are defined in convex sets and they are continuous, quasi-concave, with a negative slope, they do not cross and hence they do not have a common point.

The second element of the model is a linear constraint, presented by the line $l : a_1x_1 + a_2x_2 = b$, where b is some constant. This line links a point from the horizontal axis with a point on the vertical axis and represents a geometrical area of points, each of which represents a different combination of the arguments of the function $\varphi(x)$, x_1 and x_2, their total value being equal to the constant b.

The aim with this model is to find the vector $x^* = (x_1^*, x_2^*)$, for which the function $\varphi(x_1, x_2)$ has a common point with the constraint and in this point it reaches its maximum value. We will prove that point A (x_1^*, x_2^*) in Figure 1, represented by the vector $x^* = (x_1^*, x_2^*)$, in which the curve is tangent to the constraint *l,* is a solution to the maximization problem.

For the purposes of our analysis we introduce the **value function** $v(a,b)$, which takes the following form:

$$v(a,b) = \max_{x \geq 0} \varphi(x) \tag{1}$$
$$\text{s.t. } \langle a, x \rangle \leq b$$

By using the first order condition, the solution to this problem is the vector $x^*(a,b)$ with coordinates $x^*_k = x^*_k(a, b)$, for $k = 1, 2, \ldots, n$.

In the two-dimensional case, the vector $x^* = (x_1^*, x_2^*)$ defines the point of maximum and the function *x (a, b)*= $\begin{pmatrix} x_1(a,b) \\ x_2(a,b) \end{pmatrix}$ is a function that depends on the parameter *a* and the constraint $\langle a, x \rangle \le b$ and determines the quantity from the fist and the second variable, which are x_1 and x_2, for obtaining a maximum value of φ *(x)*, i.e. is a solution to the problem.

As the value function *v (a, b)* is monotonic with regard to *b*, then we can formulate the dual problem, i.e. for each level curve $\varphi(x) = y$ we can get the minimum value of $\langle a, x \rangle$, necessary for obtaining a certain level of *y* with a given parameter *a*.

For this purpose, we introduce the value function *g (a, y)*, which represents this relation and we formulate the problem for obtaining a minimum value of $\langle a, x \rangle$ under the constraint $\varphi(x) \geq y$, which takes the following form:

$$g(a, y) = \min_{x \geq 0} \langle a, x \rangle \qquad (2)$$

$$\text{s.t. } \varphi(x) \geq y$$

The function $h^*_k = h^*_k(a, y)$, for $k = 1, 2, \ldots, n$ is a solution to the problem. In the two-dimensional case the vector $h^* = (h_1^*, h_2^*)$ defines the point of minimum and the function *h (a, y)*= $\begin{pmatrix} h_1(a,y) \\ h_2(a,y) \end{pmatrix}$ is a function which depends on the parameter *a* and the value *y* of the function φ *(x)* and determines the necessary quantity of the first and second variables, which are h_1^* and h_2^*, for obtaining a minimum value of *g*(*a, y*), i.e. it is a solution to the problem.

The point of minimum coincides with the point of maximum, i.e. the solution to the two problems is one and the same vector. Hence, we can prove the following theorem:

Theorem 1

If the function $\varphi(x)$ is continuous and defined in a convex and compact set x $\in$ X, characterized by local non-satiation, then the optimal vector x^*, which is a solution to the maximization problem for $\varphi(x)$ determines the optimal vector x^*, which is a solution to the minimization problem of $\langle a, x \rangle$. And vice versa, the optimal vector x^*, which is a solution to the problem for minimizing $\langle a, x \rangle$ determines the optimal vector x^*, which is a solution to the problem for maximizing $\varphi(x)$. This can be formulated with the following identities:

$$v\,(a, b^*) = \max_{\langle a,x\rangle \le b^*} \varphi(\mathrm{x}) = \varphi(\mathrm{x}^*) = \varphi^*$$

and

$$g\,(a, \varphi^*) = \min_{\varphi(\mathrm{x}) \ge \varphi^*} \langle a,x\rangle = \langle a,x^*\rangle = b^*$$

Proof:

Let x^* be a vector for which the function $\varphi(x)$ has a maximum value and let $\varphi(x)=y$. We assume that there exists a vector x', at which the constraint b reaches its minimum value. Then, $\langle a,x'\rangle < \langle a,x^*\rangle$ and $\varphi(x') \geq y = \varphi(x^*)$. The local non-satiation property provides for the existence of a vector x'', close enough to x', i.e. for this vector the following inequalities are fulfilled:

$$\langle a,x''\rangle < \langle a,x^*\rangle = b \tag{3}$$

and

$$\varphi(x'') > \varphi(x') > \varphi(x^*) \tag{4}$$

From (4) it follows that we can find a vector x'', in which the function $\varphi(x)$ has a greater value. This contradicts the assumption that x^* is a vector in which we have a minimum of the value of the constraint b.

The opposite is also true. Let x^* be a vector which minimizes $\langle a,x\rangle$. Then $\langle a,x\rangle = b > 0$. We will prove that x^* maximizes $\varphi(x)$. We assume that x^* is not a solution and let x' be the vector which maximizes $\varphi(x)$. Then $\varphi(x') > \varphi(x^*)$ and $\langle a,x'\rangle = b = \langle a,x^*\rangle$. As $\langle a,x^*\rangle > 0$ and $\varphi(x)$ is a continuous function, then there exists such a number $t \in [0, 1]$, that $\langle a,tx'\rangle = t\langle a,x'\rangle < \langle a,x^*\rangle = b$ and $\varphi(tx') > \varphi(x') > \varphi(x^*)$. Hence, we have obtained a new vector tx', at which the value of b is less and thus contradicts the assumption. Therefore, the vector x^* maximizes the function $\varphi(x)$. ♦

Based on the above theorem we derive the following identities:

$$g\,(a,\ v\,(a,\ b)) = b \text{ and } v\,(a,\ g(a,\ y)) = y \tag{5}$$

and

$$x(a,\ b) \equiv h\,(a,\ v(a,\ b)) \quad\text{and}\quad h(a,\ y\,) \equiv x\,(a,\ g(a,\ y)) \tag{6}$$

From identity (5) and (6) and applying the chain rule we derive the following equation:

$$\frac{\partial x_i(a^*,b^*)}{\partial a_j}=\frac{\partial h_i(a^*,y^*)}{\partial a_j}-x_j(a^*,b^*)\frac{\partial x_i(a^*,b^*)}{\partial b}\ \text{for } i, j = 1, \ldots, n \tag{7}$$

In this equation, the derivative $\partial h_i / \partial a_j$ relates to substitution of the variables**,** which are arguments of the objective function, $x_j \times \partial x_i / \partial b$ expresses the effect of the constraint, and $\partial x_i / \partial a_j$ indicates the total effect when combining substitution and constraint. The substitution effect determines a line tangent to the curve of the function *φ(x)* and measures the impact on the h_i coordinate upon the increase in the a_j parameter in the problem for minimizing the value of *b*, and the effect of the constraint measures the impact on the x_i coordinate upon the increase in the value of the constraint in the problem for maximizing the value of the function *φ(x)*, multiplied by the x_j coordinate. The total effect $\partial x_i / \partial a_j$ determines the change of some variable, respectively ∂h_i or ∂x_i, against the change in a given parameter from the vector *a* which is found in the difference $\partial h_i / \partial a_j - x_j \times \partial x_i / \partial b$.

In the case when we have i=j, equation (7) takes the following form:

$$\frac{\partial x_j(a^*,b^*)}{\partial a_j}=\frac{\partial h_j(a^*,y^*)}{\partial a_j}-x_j(a^*,b^*)\frac{\partial x_j(a^*,b^*)}{\partial b} \tag{8}$$

From the properties of the function *h(a, y)* it follows that $\boldsymbol{\partial h_j/\partial a_j}$ has a negative sign and hence $\boldsymbol{\partial x_j/\partial a_j}$ also has a negative sign apart from the case when $\boldsymbol{\partial x_j/\partial b}$ has a negative sign, i.e. when the constraint effect is greater than the substitution effect.

Upon changes in the parameter Δa_i, for the change of the function *x* (*a*, *b*) we have:

$$\Delta x_i \equiv \frac{\partial x_i(a,b)}{\partial a_1}\Delta a_1+\frac{\partial x_i(a,b)}{\partial a_2}\Delta a_2+\ldots+\frac{\partial x_i(a,b)}{\partial a_n}\Delta a_n$$

Using equation (7) we obtain:

$$\partial x_i(a,b)\Delta a_j=\frac{\partial h_i(a,v(a,b))}{\partial a_j}\Delta a_j-\frac{\partial x_i(a,b)}{\partial a_j}\frac{x_j(a,b)}{\partial b}\Delta a_i \tag{9}$$

The value of the function *x*(*a*, *b*) changes with the change of the parameter that influences the constraint, which means that the optimal vector changes and moves to a higher level curve that defines a greater value of the function *φ(x)*, i.e. what we observe is the constraint effect.

Depending on the changes in the value of the parameter *a* and the parameter *b,* the variables which comprise the optimal vector x^* can be classified as:

1. *normal variables* - variables for which at a fixed value of the parameter *a* the increase in the value of *b* leads to an increase in the value of the function *x (a, b),* i.e. $\frac{\partial x_j}{\partial b} > 0$. The value of the function *x(a, b)*, consisting of *normal* variables, decreases upon the increase in the value of the parameter *a*, and vice versa. Such variables we shall also refer to as *ordinary*.

2. In cases when upon the presence of two variables between which a choice is being made, with the increase in the constraint and at a fixed value of the parameter *a,* the value of one variable increases proportionately more than the value of the other variable, then we define the first variable as *luxurious* and the second variable as *necessary.* This result implies that the coefficient of proportionality for the necessary variable is less than the coefficient of proportionality for the luxurious variable.

3. However, if $\frac{\partial x_j}{\partial b} < 0$, then $\frac{\partial x_j}{\partial a_j}$ is determined by the negative substitution effect and the positive constraint effect. Hence, the derivative $\frac{\partial x_j}{\partial a_j}$ can be either positive or negative. If the constraint effect is greater than the substitution effect then:

$$\frac{\partial x_j}{\partial a_j} = \frac{\partial h_j}{\partial a_j} - \frac{\partial x_j}{\partial b} x_j(a,b) > 0\,,$$

which means that the value of the function *x (a, b)* for the variable *j* has increased with the increase in the value of the parameter *a* and $\frac{\partial x_j}{\partial a_j} > 0$.

Also the opposite is true – the value of *x (a, b)* for the variable *j* has decreased with the decrease in the value of the parameter *a.* This variable we can refer to as a *Giffen variable*. The Giffen variables are also *inferior variables* as for them it is true that $\frac{\partial x_j}{\partial b} < 0$, which means that with the increase in the constraint the value of *x (a, b)* also decreases.

4. If the constraint effect is less than the substitution effect then $\frac{\partial x_j}{\partial a_j} < 0$, which means that the variable is both *ordinary* and *inferior*. Inferior variable is that variable for which the value of the function *x(a, b)* decreases upon the increase in the value of the constraint *b*.

5. Variables for which $\frac{\partial h_i}{\partial a_j} = \frac{\partial h_j}{\partial a_i} > 0$ are *substitutes,* and variables for which $\frac{\partial h_i}{\partial a_j} = \frac{\partial h_j}{\partial a_i} < 0$ are *complements.*

3. Applications of the model in the choice of labor supply

The dual problem and its solution have been discussed by Menezes and Wang (2005), who use the Slutsky equation in their analysis of the change in the optimal supply of labor under the conditions of wage uncertainty and risk. In their model $L, Y \geq 0$ are respectively the quantity of labor and income and they are arguments of the von Newman-Morgenstern utility function $u(L,Y)$, which is decreasing in L, increasing in Y, concave in (L, Y) and thrice continuously differentiable. The income of the consumer is defined with the equation: $Y = Y_0 + wL$, where Y_0 is his non-labor income and the price of labor w is a positive, random variable, or $w = \overline{w} + \beta z$. In this equation $\overline{w} = Ew$ is the expected wage rate, z is a neutral, random variable, and β is a positive vector, which can be used as a spreading risk parameter.

Menez and Wang provide another application of problem (1) when studying the maximization of the individual's labor supply function:

$$v(L,\overline{Y},\beta) = \max_{0\leq L\leq L_0} Eu(L,\overline{Y} + \beta zL) \qquad (10)$$
$$\text{s.t. } Y = \overline{Y} + \beta zL$$

where $\overline{Y} = Y_0 + \overline{w}L$ is the expected level of income. The function $v(L,\overline{Y},\beta)$, which they refer to as the „derived utility function' is the well-known indirect utility function.

Menezes and Wang further formulate the dual problem through the cost function:

$$I(\overline{w},v^0,\beta) = \min_{0\leq L\leq L_0} \overline{Y} - \overline{w}L \qquad (11)$$
$$\text{s.t. } v(L,\overline{Y},\beta) = v^o$$

The function $I(\overline{w},v^0,\beta) \equiv \overline{Y}^C(\overline{w},v^0,\beta) - \overline{w}L^C(\overline{w},v^0,\beta)$ in their analysis is used to determine the minimum non-labor income, required for achieving expected utility level v^0.

Menezes and Wang claim that the supply of labor $L^U(\cdot)$ and the compensated supply $L^C(\cdot)$ coincide, when the non-labor income is represented with the equation $Y_0 = I(\overline{w},v^0,\beta)$, which they determine with the identity $L^U(\overline{w},I(\overline{w},v^0,\beta),\beta) \equiv L^C(\overline{w},v^0,\beta)$. The authors make this proposition solely on the basis of the solution of problems (10) and (11). In the context of our model, the proposed relation can be easily proven by applying Theorem 1 from Section 2.

Menezes and Wang further formulate the Slutsky equation:

$$\frac{\partial L^U(\cdot)}{\partial s} = \frac{\partial L^C(\cdot)}{\partial s} - \frac{\partial L^U(\cdot)}{\partial Y_0} \times \frac{\partial I(\cdot)}{\partial s} \tag{12}$$

where s is equal either to the parameter $\overline{w}$ or to β, $-\left[\partial L^U / \partial Y_0\right] \times \left[\partial I / \partial s\right]$ represents the income effect, $\partial L^C / \partial s$ is the substitution effect, and $\partial L^U / \partial s$ is the total effect. Considering the interaction of these two effects, Menezes and Wang (2005) prove that under conditions of wage uncertainty and risk, the income effect is positive (negative) depending on whether leisure is a normal (inferior) good. According to them, leisure is normal (inferior) good only when $\partial L^U / \partial Y_0 < 0 (> 0)$.

However, our model proves that the Slutsky equation can be applied in a more general analysis of the changes in the optimal labor choice. In the general analysis of changes in the optimal decision between labor and leisure, we can additionally extend the model of Menezes and Wang by applying the classification form Section 2. Thus, depending on the income and the substitution effects, leisure can be analyzed not only as *normal* or *inferior* good, but also as *ordinary, necessary, luxurious* or a *Giffen good*.

4. Influence of taxes on the optimal choice between labor and leisure

A standard Lindahl's optimization model will be analyzed, where there is an aggregate quantity of labor L, an aggregate quantity of leisure d, and n is the number of workers within a given community. It will further be assumed that labor income is constant and the community consists of equal income groups.

We shall apply problem (1) and thus formulate the utility maximization problem for the choice between leisure and labor, which takes the following form:

$$v(\tau, I) = \max_{L \geq 0, d \geq 0} u(L, d) \tag{13}$$

$$\text{s.t.} \quad \tau_1 L + \tau_2 d = I$$

where the objective function $u(L,d)$ is the utility function which represents the utility of the workers, L is the total aggregate quantity of labor, d is the total aggregate quantity of leisure, τ_2 is the value of leisure, τ_1 is the price of labor in the form of wages and salaries, and I is the total amount of the budget constraint or income. We assume that τ_2 is constant, i.e. we isolate any possible changes in the prices of the private goods found in the consumption bundle and also of those private goods which are outside it. Hence, the function $v(\tau, I)$ is the indirect utility function, which describes the preferences in the choice between labor and leisure. The solution to problem (23) is the Marshallean demand function $x^*(\tau, I)$, which describes the choice between private and local public goods.

Furthermore, by applying problem (2) we shall formulate the dual problem and continue the analysis by observing the expenditure minimization problem in the process of consumption of public and private goods, which takes the following form:

$$e(\tau,u) = \min_{L\geq 0, d\geq 0} \tau_1 L + \tau_2 d \qquad (14)$$
$$\text{s.t.}\, u(L,d) \geq u$$

where the function $e(\tau,u)$ is the expenditure function. The solution to this problem is the Hicksian demand function $h^*(\tau,u)$, which represents the choice for public and private goods supplied in the community. By applying theorem 1 we derive the following equations:

$$v(\tau, I^*) = \max_{\tau_1 L + \tau_2 d \leq I^*} u(L,d) = u(L^*, d^*) = u^* \qquad (15)$$

and

$$e(\tau, u^*) = \min_{u(L,d)\geq u^*} \tau_1 L + \tau_2 d = \tau_1 L^* + \tau_2 d^* = I^* \qquad (16)$$

From (15) and (16), the following identities are valid:

$$e(\tau, v(\tau, I)) = I \text{ and } v(\tau, e(\tau,u)) = u \qquad (17)$$

and

$$x(\tau, I) \equiv h(\tau, v(\tau, I)) \text{ and } h(\tau,u) \equiv x(\tau, e(\tau,u)) \qquad (18)$$

If we fix the price for labor supply τ_1 as constant, then it is the value of the respective tax rates which influences the choice of a given individual within a community and determines the quantity of labor an leisure. The equilibrium theory suggests that with the increase in the value of taxes, labor becomes either inferior or a Giffen good and vice versa, with the decrease in the value the tax rates, leisure becomes Giffen or inferior good and labor either remains of the type before the change of the tax rate or it turns into normal and even in some cases luxury good.

The maximization problem takes the following form:

$$v\ (\psi, B) = \max_{L\geq 0, d\geq 0} u(L,d) \qquad (19)$$
$$\text{s.t. } \psi_1 L + \psi_2 d = B$$

where the objective function $u(L,d)$ is the utility function, ψ_1 is the amount of taxes paid for labor supply and ψ_2 is the price of leisure, which is a constant variable, and B is the budget spent on the financing of taxes and leisure. Hence, the solution to problem (19) is the Marshallean demand function $x^*(\psi, B)$, which expresses the optimal choice of two types of goods.

We can now formulate the dual problem:

$$e(\psi,u) = \min_{L\geq 0, d\geq 0} \psi_1 L + \psi_2 d \qquad (20)$$
$$\text{s.t.} \quad u(L,d) \geq u$$

where the function $e(\psi,u)$ is the expenditure function and the solution is the Hicksian demand function $h^*(\psi,u)$, which is the vector of the choice between labor and leisure. Based on Theorem 1 and identities (5) and (6) it is obvious that $x^*(\psi, B) \equiv h^*(\psi,u)$.

Therefore, when $\psi_2 = 0$, the Slutsky equation (7) for model (19) takes the form:

$$\frac{\partial x_j(\psi, B)}{\partial \psi} = \frac{\partial x_j(\psi, B)}{\partial B} x_i(\psi, B) \qquad (21)$$

Depending on the decision of the government with regard to the amount of tax rates imposed on labor, as a good it can be classified as *normal* when the tax is equally distributed and hence the quantity of labor and leisure increases in the same proportion.

Then, from the Slutsky equation (7) and by applying the classification from the theoretical model in Section 2, labor and leisure can be classified as: *normal, ordinary, luxurious, inferior, Giffen goods, substitutes and complements.*

5. Conclusion

In this paper a theoretical optimization model was applied in order to analyse the choice between labor and leisure through the solution of a pair of problems – the maximization problem and the minimization problem. By introducing the dual problem and on the basis of our Theorem 1, it was proved that the solution to the maximization problem is also a solution to the minimization problem. In our theoretical model a universal equation was derived, similar to the familiar Slutsky equation from the consumption theory.

Further on, an application of the model in the choice of labor supply was commented, derived from existing economic literature, but the distinction that the Slutsky equation can be applied in a more general analysis of the changes in optimal labor choice was made clear. Our contribution to this application of the model is the claim that depending on the income and substitution effects, the arguments of the objective function (labor and leisure) can be classified using a general classification of the goods, i.e. they can be analyzed as *normal, ordinary, necessary, luxury, Giffen* or *inferior goods*.

To support these arguments, the influence of tax rates on the optimal choice between labor and leisure was analysed. Again, by applying the general theoretical model, it was demonstrated that both goods can be studied and classified following the classification from the theoretical model.

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