IJBESAR

International Journal of Business and Economic Sciences Applied Research
8(1): 113-120
http://ijbesar.teiemt.gr

On the complex dynamics of a bounded rational monopolist model

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Abstract

Purpose *– The main purpose of this paper is to investigate the dynamic behavior of a bounded rational monopolist with a general nonlinear demand and quadratic cost functions reflecting diseconomies of scales.*
Design/methodology/approach *– We suppose that locally the monopoly firm uses a gradient mechanism and looks at how the rate of growth of the quantity affects the variation of profits.*
Findings *– We prove that the nonzero steady state is exactly the level of production that maximizes profits, as can be seen in the classic microeconomic theory. However, complex dynamics can arise.*
Research limitations/implications *– For some values of a parameter there is a locally stable equilibrium which is the value that maximizes the profit function. Increasing these values, the equilibrium becomes unstable, through period-doubling bifurcation.*
Originality/value *– The result indicates that a limited reaction of the monopolist to changes in profits can stabilize the quantity produced. On the other hand turbulences in the market are generated by an overreaction.*

Keywords: Monopoly, Difference Equation, Equilibrium, Stability, Chaotic Behavior

JEL Classification: C61, C62, D42

1. Introduction

The canonical approach of the monopoly theory is essentially static and the monopolist has full rationality: both perfect computational ability and large informational set in such a way that she can determine both quantity and price to maximize profits. However, in the real market producers do not know the entire demand function, though it is possible that they have a perfect knowledge of technology, represented by the cost function. Hence, it is more likely that firms employ some local estimate of the demand. This issue has been previously analyzed by Baumol and Quandt, 1964; Puu, 1995; Naimzada and Ricchiuti, 2008, Askar, 2013. Naimzada and Ricchiuti evaluate a discrete time dynamic model with a cubic demand function without an inflexion point and linear cost function.

In recent years, many researchers have demonstrated that economic agents may not be fully rational. Even if one tries to perform things correctly, it is important to utilize simple rules previously tested (Kahneman et al., 1986; Naimzada and Ricchiuti, 2008). Efforts have been made to model bounded rationality to different economic areas: oligopoly games (Agiza, Elsadany, 2003; Bischi et al., 2007); financial markets (Hommes, 2006); macroeconomic models such as multiplier-accelerator framework (Westerhoff, 2006). In particular, difference equations have been employed extensively to represent these economic phenomena (Elaydi, 2005; Sedaghat, 2003).

In this paper, the equilibrium state of a bounded rational monopolist model is studied. It is assumed a general demand and quadratic cost functions and that locally the monopoly firm uses a gradient mechanism and looks at how the rate of growth of the quantity affects the variation of profits. We show that complex dynamics can arise and the stability of the nonzero equilibrium state is discussed. The complex dynamics, bifurcations and chaos are displayed by computing numerically Lyapunov numbers and sensitive dependence on initial conditions.

2. The model

The inverse demand function has a general form, it is downward sloping and concave:

$$p = a - bq^n, n \in Z, n > 2 \quad (1)$$

where p indicates commodity price, q indicates the quantity demanded and a and b are positive constants. The downward sloping is guaranteed if:

$$\frac{dp}{dq} = -nbq^{n-1} < 0 \quad (2)$$

that is if b>0.

The quantity produced, q, is positive and non-negative prices are achieved if

$$q < \sqrt[n]{\frac{a}{b}} \qquad (3)$$

We suppose that the cost function is quadratic

$$C(q)=cq^2 \qquad (4)$$

Moreover, we assume the general principle of setting price above marginal cost, p - c > 0, for each non negative q; that is, a > c. The main aim of the firm is to maximize the following profit function

$$\Pi(q)=(a-bq^n)q-cq^2 \qquad (5)$$

This function is concave and gives the following first order condition:

$$\frac{d\Pi}{dq}=a-2cq-(n+1)bq^{n-1}=0 \qquad (6)$$

The marginal profit is strictly decreasing with range in the interval $(-\infty,a]$, therefore Eq. (6) has a unique solution q^* in this interval and the profit has a maximum at q^*. If $\Pi(q^*)>0$ a positive equilibrium production is guaranteed.

To achieve increasing profits, it is assumed that locally the monopoly firm, using a gradient mechanism, looks at how the rate of growth of the quantity affects the variation of profits. A positive (negative) variation of profits will induce the monopolist to change the quantity in the same (opposite) direction from that of the previous period. No changes will occur if profits are constant. This mechanism can be represented as follows:

$$\frac{q(t+1)-q(t)}{q(t)}=k\frac{d\Pi}{dq(t)},\ t=0,1,2,... \qquad (7)$$

where k>0 is the speed of adjustment to misalignments. Substituting Eq. (6) in (7), we obtain the following one-dimensional nonlinear difference equation:

$$q(t+1)=q(t)+kq(t)\cdot[a-2cq(t)-(n+1)bq^n(t)] \qquad (8)$$

3. Dynamical Analysis

3.1 Equilibria and stability

If

$$f(q)=q+kq[a-2cq-(n+1)bq^n] \qquad (9)$$

the fixed points of Eq. (8) are the solutions of the equation $f(q)=q$, and then the nonzero fixed point is the solution q^* of Eq. (6). Since

$$\frac{df}{dq}\left(q^*\right)=1+kq^*\frac{d^2\Pi}{\partial q^2}(q^*) \qquad (10)$$

the steady state is locally stable if:

$$\left|1+kq^*\Pi''(q^*)\right|<1 \qquad (11)$$

or, equivalently,

$$0<k<\frac{2}{-q^*\Pi''(q^*)} \qquad (12)$$

It follows that:

Proposition. Map (8) has a unique nonzero steady state $q(t) = q^*$ which is exactly the quantity that maximizes profits. It is locally stable if $0 < k < \frac{2}{-q^* \Pi''(q^*)}$

3.2. Numerical simulations

The previous result indicates that a limited reaction of the monopolist to changes in profits can stabilize the quantity produced. On the other hand turbulences in the market are generated by an overreaction. To shed some light on what really happens in the market we employ a numerical analysis. Fixing the other parameters of the model as follows:

$a = 4, b = 0.6, c = 0.5$, then, for $n = 6$, $q^* \approx 0.948$, $k^* \simeq 0.104$. The dynamic map (8) satisfies the canonical conditions required for the flip bifurcation (Abraham et al., 1997) and there is a period doubling bifurcation if $k = k^*$. When $k < k^*$ the fixed point is attracting, when $k > k^*$ it is repelling. Therefore, there is a change in the nature of dynamics when $k = k^*$, a unique asympto-tically stable period two-cycle arises.

We graphically show how the behavior of the map (8) changes for different values of the reaction coefficient, k. (Kulenovic, Merino, 2002).

In Figure 1, we show the map (8) when k = 0.09. From Eq. (13), the steady state is asymptotically stable.

In Figure 2, we show the particular set of parameters that determines a period two-cycle, actually, with k = 0.12. Further growth of k leads the attractor to follow a typical route of flip bifurcations in complex price dynamics: a sequence of flip bifurcations generate a sequence of attracting cycles in period 2^n, which are followed by the creation of a chaotic attractor.

In Figure 3, a cycle of period four is shown. To clarify the dynamics depending on k, we have reported a bifurcation diagram in Figure 4. It shows different values of quantity for different values of k, particularly between 0 and 0.18. It is easily illustrated that we move from stability through a sequence of a period doubling bifurcations to chaos.

In Figure 4 are represented also the Lyapunov numbers of the orbit of 0.01, for k = 0.17, versus the number of iterations of the map (8). If the Lyapunov number is greater of 1, one has evidence for chaos. To demonstrate the sensitivity to initial conditions of Eq. (6), we compute two orbits (100 iterations of the map) with initial points $q_0 = 0.01$ and $q_0 + 0.0001$, respectively.

The results are shown in Figure 5. At the beginning the time series are indistinguishable; but after a number of iterations, the difference between them builds up rapidly.

Figure 2: Cycle of period 2, for a =4, b=0.6, c= 0.5, n=6 and k= 0.12

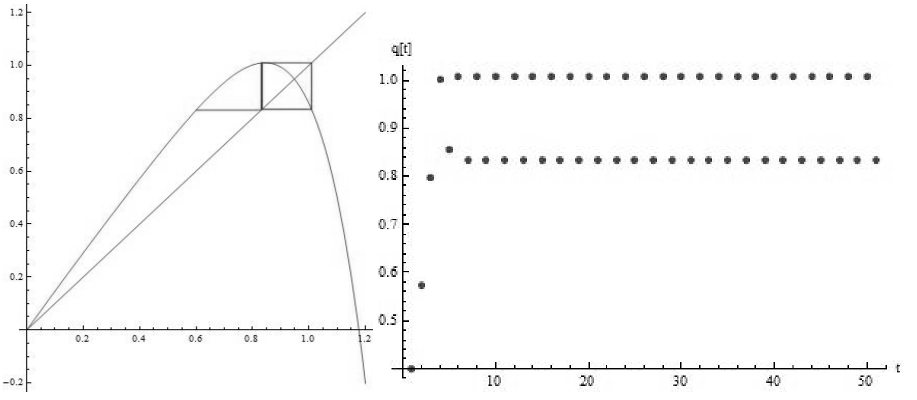


Figure 3: Cycle of period 4, for a =4, b=0.6, c= 0.5, n=6 and k= 0.134

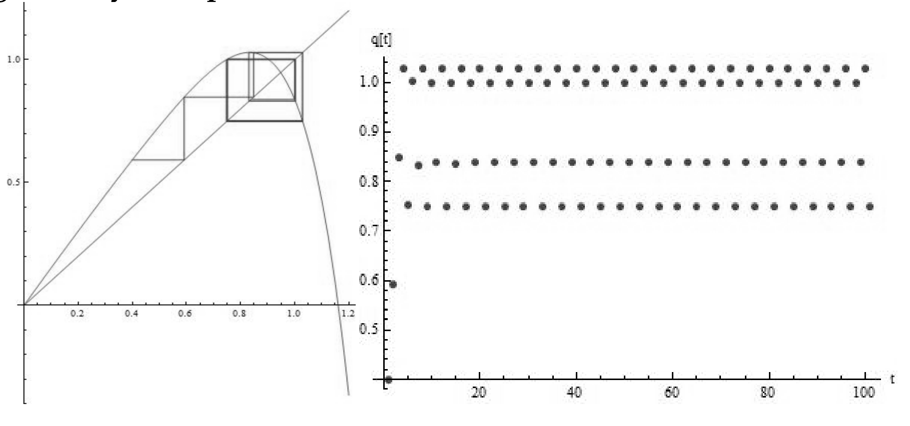


Figure 4: For n=6, bifurcation diagram with respect to the parameter k against variable q, for q_0=0.01 and 550 iterations of the map (8) (left) and Lyapunov numbers of the orbit of 0.01, for k =0.17, versus the number of iterations of the map (8) (right).

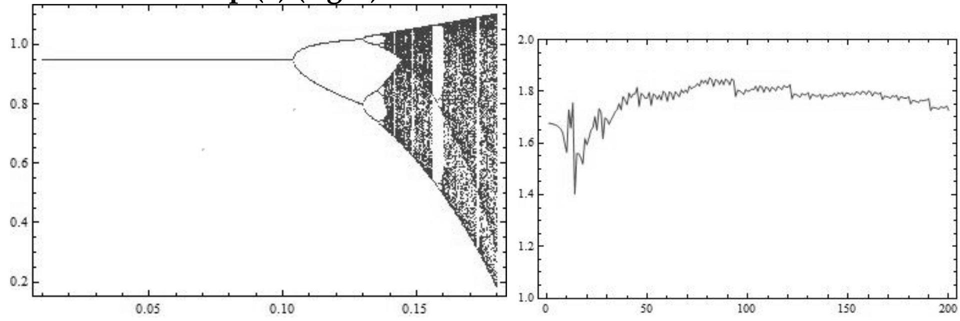
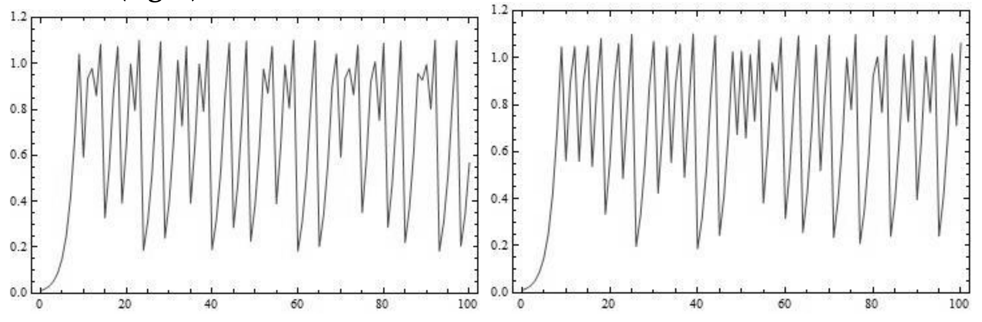


Figure 5: For n = 6, sensitive dependence on initials conditions: q plotted against the time, parameter value k=0.17 and initial condition q_0 = 0.01 (left), q_0 = 0.0101 (right).



4. Conclusion

In this paper, we have analyzed the effects on the equilibrium of a monopoly when the monopolist has bounded rationality. We employ a discrete time dynamical model such as that used by Askar 2013; however, we use a quadratic cost function and we suppose that locally the monopoly firm uses a gradient mechanism, looks at how the rate of growth of the quantity affects the variation of profits. We prove that for some values of a parameter there is a locally stable equilibrium which is the value that maximizes the profit function. Increasing these values, the equilibrium becomes unstable, through period-doubling bifurcation. The complex dynamics, bifurcations and chaos are displayed by computing numerically Lyapunov numbers and sensitive dependence on initial conditions. The result indicates that a limited reaction of the monopolist to changes in profits can stabilize the quantity produced. On the other hand turbulences in the market are generated by an overreaction. The case of demand and cost functions of a more general form and comparing the results is left for future research.

Acknowledgements

The author thanks the anonymous referees for interesting remarks and useful suggestions.

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